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Time-Inconsistent Discounting and the Friedman Rule: The Role of Non-Unitary Discounting*

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Abstract

We examine the optimality of the Friedman rule by considering recent developments in behavioral economics. We construct a simple macroeconomic model where agents discount consumption and leisure at different rates. We also consider a standard exponential discounting model and a hyperbolic discounting model, assuming that the same discounting applies to both consumption and leisure. Money is introduced via a cash-in-advance constraint. Although the three models are observationally equivalent, they provide different policy implications. The Friedman rule is optimal in the latter two models, whereas it is not optimal in the first model when agents discount consumption at a higher rate than leisure.

Keywords: Non-unitary discount rate, Hyperbolic discounting, Exponential discounting, Friedman rule, Optimal inflation

JEL Classification Numbers: E5, E71

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1 Introduction

The purpose of this study is to examine how recent evidences from psychology and behavioral economics affect the optimality of the Friedman rule in a simple macroeconomic model. Since Friedman (1969), many studies have examined the optimality of the Friedman rule, which sets the nominal interest rate at zero. Using standard models, Chari and Kehoe (1999) show that the Friedman rule is optimal in the long run.[1] In a different context, psychology and behavioral economics have revealed that standard economic models often fail to describe people's preferences and behaviors. Naturally, a policy recommendation based on a model that cannot accurately capture people's behaviors could be misguided. Therefore, when assessing economic policies such as the Friedman rule, it is quite important to use a model that is consistent with the recent evidences from psychology and behavioral economics.

For our purpose, we consider the evidence that people use different discount rates for different sources of utility (see Subsection 1.1). Motivated by this evidence, Hori and Futagami (2017) develop a non-monetary model in which agents discount their utilities from consumption and leisure at different rates; we call this a non-unitary discount rate model. Except for the difference in discount rates between consumption and leisure, the model of Hori and Futagami (2017) is standard. The present study introduces money into the model of Hori and Futagami (2017) through a cash-in-advance (CIA) constraint on consumption. In addition, assuming that the same discounting applies to both consumption and leisure, we also consider (i) a standard model with exponential discounting and (ii) a model with time-variable discounting that includes hyperbolic discounting. We then examine how the different assumptions on time-discounting affect the optimality of the Friedman rule.

We first show that in the non-unitary discount rate model, where agents discount consumption and leisure at different rates, preferences are time-inconsistent. It is well known that time-variable discounting, such as hyperbolic discounting, also gives rise to time-inconsistency. However, the sources of time-inconsistency in the two models are different. In the non-unitary discount rate model, time-inconsistency arises because the marginal rate of substitution between consumption and leisure is time varying. In contrast, in the time-variable discounting model, the marginal rate of substitution between consumption and leisure is constant over time.

To formally solve our non-unitary discount rate model, we follow the theoretical literature of time-inconsistency and consider an individual as composed of a sequence of autonomous decision makers.[2] The choices of each decision maker (self) can be considered the outcome of an intrapersonal game. We show that under log utility, the non-unitary discount rate model becomes observationally equivalent to the standard exponential discounting model and the model with time-variable discounting that includes hyperbolic discounting.

In spite of observational equivalence, the three models provide different implications for the Friedman rule. In the standard exponential discounting model and time-variable discounting model, the Friedman rule is optimal. In contrast, in the non-unitary discount model, the Friedman rule is not optimal as long as agents discount the utility of future consumption at a higher rate than the utility of future leisure. Moreover, we show that the optimal inflation rate increases as the financial market develops.

The intuition behind this result is as follows. The fact that agents discount their future consumption more than their future leisure means that today's self cares less about the future consumption than future leisure, whereas future selves care about their own consumption as

[1]We discuss theoretical literature in Subsection 1.2.

[2]See Peleg and Yaari (1973), Goldman (1980), Laibson (1996, 1997), and Luttmer and Mariotti (2003), for example.

much as they do about their leisure. Thus, future selves consume more than that today's self prefers. Higher inflation tightens the CIA constraint and reduces the consumption of future selves, resulting in a positive effect on the welfare of today's self. Since there is a positive relation between inflation and the nominal interest rate through the Fisher equation, the Friedman rule is not optimal.

Our results suggest the following points. First, different assumptions about time discounting can affect the optimality of the Friedman rule. Thus, incorporating evidences from psychology and behavioral economics is important when assessing various economic policies such as the Friedman rule. Second, time-inconsistency of preferences does not necessarily affect the optimality of the Friedman rule. If hyperbolic discounting leads to time-inconsistency, the Friedman rule is optimal. However, if differences in discount rates lead to time-inconsistency, the Friedman rule is not optimal. Thus, we need to consider the source of time-inconsistency. Third, even if two economies are observationally equivalent to each other, the optimal policies in each economy could differ.

1.1 Relation to Psychology and Behavioral Economics

The literature from psychology and behavioral economics provide evidences supporting our assumption that people use different discount rates for different utility sources. Since Hori and Futagami (2017) provide a detailed review of the literature, we present only a short summary here.[3]

Frederick et al. (2002) criticize the standard assumption of economics that people apply a single discount rate to their utility from different sources. They argue that a person who smokes heavily may carefully study the returns of various retirement packages. He may discount the disutility of poor future health at a higher rate. At the same time, a careful study of the returns of various retirement packages suggests a much lower discount rate for the utility of consumption after retirement.

In fact, recent studies in psychology and behavioral economics provide empirical and experimental evidences that people discount their utility from different sources at different rates. The term "domain effect" is used to refer to the finding that the discount rates for the different domains may differ. Through a series of experiments, Chapman and her coauthors showed that people use different discount rates for money and health (Chapman, 1996; Chapman et al., 1999; and Chapman et al., 2001). Using survey data from rural Uganda, Ubfal (2016) found that the discount rates for entertainment and cell phone airtime are lower than the discount rate for money, while those for food are higher than the discount rate for money.

The domain effect has been observed for money and time/effort. Soman (1998) conducted an experiment to study the discounting of money and time/effort. He found that people discount future time/effort at different rates from the perspective of future money (see also Soman et al., 2005). Soman (2004) and Zauberman and Lynch (2005) also provided similar results.

These studies motivate us to construct a model where agents use different discount rates for time and consumption. Section 2 presents a model where agents apply different discount rates for consumption and leisure.

[3] See the subsection titled "Related Literature from Psychology and Behavioral Economics" in the Introduction of Hori and Futagami (2017).

1.2 Relation to Theoretical Literature

Friedman (1969) states that the rate of nominal interest should be set at zero because the cost of printing money is zero. The validity of this simple rule has been debated. Grandmont and Younes (1973) show its validity by using a simple macroeconomic model with a CIA constraint. Phelps (1973) argues that if governments must rely on the distortionary taxes to collect revenue, they must impose the inflation tax to correct the distortion, which could result in the rule failing to hold. Other arguments attribute the validity of the rule to the shopping time function (Woodford, 1990). Mulligan and Sala-i-Martin (1997) summarize these arguments and further calibrate their model and show that the optimal nominal interest rate should be less than 1 percent. Some authors, such as Freeman (1993) and Bhattacharya et al. (2005), use overlapping generations models to examine the optimality of the Friedman rule.[4]

One limitation of these studies is that they pay little attention to recent developments in psychology and behavioral economics. For example, they assume constant discounting, which is inconsistent with evidences from psychology and behavioral economics. So far, only a few studies have considered the findings of behavioral economics when examining the optimality of the Friedman rule. Graham and Snower (2008, 2013) incorporate hyperbolic discounting into the New Keynesian models with nominal wage rigidity and elastic labor supply. They argue that agents with hyperbolic discounting attach much less weight to the disutility from future labor supply than agents with standard exponential discounting. If nominal wage is sticky, inflation decreases the future real wage and thus increases future labor demand. Then, agents with hyperbolic discounting tend to supply more labor since they discount the future disutility of labor heavily. This yields a positive relation between inflation and labor supply. Graham and Snower (2013) show that because of this positive relation, the Friedman rule is not optimal. Here, the following point should be noted. In their model, hyperbolic discounting generates time-inconsistency. However, time-inconsistency itself is not the source of non-optimality of the Friedman rule. They insist that heavy discounting of the disutility of labor, coupled with nominal wage rigidity, creates a positive correlation between inflation and labor supply, and this renders the Friedman rule non-optimal.

In contrast to Graham and Snower (2013), we assume flexible prices. We show that without price stickiness, the Friedman rule is optimal even with hyperbolic discounting. We also show that if the agents' discount rate for future consumption is higher than that for future leisure, the Friedman rule is not optimal even without price stickiness. This is because time-inconsistency causes inefficiency with regard to consumption and leisure choice in our model.

Hiraguchi (2016) examines how the temptation utility affects optimality of the Friedman rule. To this end, he incorporates Gul-Pesendorfer preferences into a search-theoretic model proposed by Lagos and Wright (2005).[5] Hiraguchi (2016) shows that a positive nominal interest rate reduces temptation, and therefore the Friedman rule is not optimal. Although this is similar to our result, the mechanism is different because Gul-Pesendorfer preferences do not generate time-inconsistency.

This study is not the first one that assumes different discount rates for different utility sources. Banerjee and Mullainathan (2010) consider a simple two-period model consisting of two types of goods: temptation goods that generate utility from today's consumption, but not from tomorrow's consumption, and the usual goods that generate utilities from both today's and tomorrow's consumption. They use their model to explain some of the puzzling behaviors

[4] See also Abel (1987) and Gavhari (1988).

[5] Gul and Pesendorfer (2001) propose a model where individuals face the problem of self-control when they face temptation goods.

of the poor. Hori and Futagami (2017) examine the effects of tax policies in a model where agents apply different discount rates to consumption and leisure, and show that the optimal tax policies are different from those obtained in a model in which agents apply a single discount rate (function) to consumption and leisure. Both studies do not consider the monetary policy. This study is the first one that examines the optimality of the Friedman rule assuming that agents discount their utility from consumption and leisure at different rates.

Organization of the Paper

In Section 2, we present our basic model. Section 3 derives a solution for the intrapersonal game. Using a simple general equilibrium model, Section 4 examines the optimal monetary policy. Section 5 compares the results of our non-unitary discount rate model with those of a model with a discount rate that varies with time distance. Concluding remarks are presented in Section 6.

2 The Model

The population size of the economy is one. We consider an infinitely-lived representative agent endowed with one unit of time at each moment of time that is allocated to labor or leisure. The preference of the agent is given by

$$U_t = \int_t^{+\infty} \left\{ u_1(c_v)e^{-\rho_c(v-t)} + u_2(1-l_v)e^{-\rho_l(v-t)} \right\} dv, \tag{1}$$

where $c_v \geq 0$ is the consumption level at time v and $l_v \in [0,1]$ is the time allocated to labor supply at time v. $u_1(c_v)$ and $u_2(1-l_v)$ represent the instantaneous utility derived from consumption and leisure at time v, respectively. Functions $u_1(\cdot)$ and $u_2(\cdot)$ are twice differentiable and satisfy $u_i'(\cdot) > 0$ and $u_i''(\cdot) < 0$ ($i = 1$, or 2). Parameters $\rho_c > 0$ and $\rho_l > 0$ are subjective discount rates for consumption and leisure, respectively. We allow the case where ρ_c is not equal to ρ_l, meaning that the agent discounts utility from different sources at different rates. When ρ_c is (not) equal to ρ_l, we call it a (non-)unitary discount rate case.[6] The unitary discount rate case gives a standard exponential discounting model where consumption and leisure are discounted at the same rate.

We denote the price of consumption good and nominal cash holdings of the agent at time t as p_t and M_t, respectively. A fraction of purchase of the consumption good, $\eta \in [0,1]$, must be financed by cash. As the financial market develops, η decreases. M_t must satisfy the CIA constraint $M_t \geq \eta p_t c_t$, or equivalently

$$m_v \geq \eta c_v, \tag{2}$$

where $m_v \equiv M_v/p_v$. The budget constraint is given by

$$\dot{a}_v = r_v a_v - (r_v + \pi_v)m_v + w_v l_v - c_v + T_v, \tag{3}$$

where r_v is the real interest rate, w_v is the wage rate, $\pi_v \equiv \dot{p}_v/p_v$ is the inflation rate, and a_t is equal to b_v+m_v, where b_v represents asset holdings other than cash. The lump-sum transfer from government is T_v. We assume that at any moment of time, agents can allocate their portfolio

[6]If consumption is tempting in the spirit of Banerjee and Mullainathan (2010), (1) can be rewritten as $U_t = u_1(c_t) + \int_t^{+\infty} u_2(1-l_v)e^{-\rho_l(v-t)}dv$.

between cash and other assets without any costs.[7]

As for the government's money supply behavior, we assume a helicopter drop of money. The monetary authority issues nominal money at a constant growth rate, $\epsilon \equiv \dot{M}_t/M_t$. The newly created money is transferred to agents in a lump-sum manner, $T_t = \epsilon M_t/p_t$.

Non-Unitary Discount Rate and Time-Inconsistency

This subsection observes that the preference represented by (1) raises the problem of time-inconsistency. For simplicity, we assume that the CIA constraint (2) binds, which is true in equilibrium, as we will see later. Then, the budget constraint (3) can be written as

$$\dot{a}_v = r_v a_v + w_v l_v - \{1 + \eta(r_v + \pi_v)\}c_v + T_v. \tag{4}$$

Before providing a formal solution in the next section, we consider the case where at time t, the agent chooses sequence $\{c_v,\ l_v,\ a_v\}_{v=t}^{\infty}$ without considering the possibility that she reconsiders her choices at some future time. As in Hori and Futagami (2017), we first maximize (1) subject to (4) by setting the present value Hamiltonian as $H_v = u_1(c_v)e^{-\rho_c(v-t)} + u_2(1-l_v)e^{-\rho_l(v-t)} + \psi_v[r_v a_v + w_v l_v - \{1+\eta(r_v+\pi_v)\}c_v + T_v]$, where $v(\geq t)$ indicates future selves and ψ_v is the costate variable associated with (4). From the first-order conditions, we obtain

$$\frac{u_2'(1-l_v)}{u_1'(c_v)}e^{-(\rho_l-\rho_c)(v-t)} = \frac{w_v}{1+\eta(r_v+\pi_v)}. \tag{5}$$

At time t, the agent plans to consume goods and supply labor according to (5) at time $v(> t)$. The left-hand side of (5) shows that as long as $\rho_c \neq \rho_l$, the marginal rate of substitution between consumption and leisure is time varying.

However, if she maximizes her utility once again at time $v(> t)$, we obtain

$$\frac{u_2'(1-l_v)}{u_1'(c_v)} = \frac{w_v}{1+\eta(r_v+\pi_v)}. \tag{6}$$

In the unitary discount rate case ($\rho_c = \rho_l$), (5) becomes exactly the same as (6). However, in the non-unitary discount rate case ($\rho_c \neq \rho_l$), (5) is different from (6), suggesting that the preferences of the agent are time-inconsistent. The time-varying marginal rate of substitution between consumption and leisure in (5) is the source of time-inconsistency.

3 Generalized Euler Equation in a Monetary Economy

To provide a formal solution of our model, we consider the agent as composed of a sequence of autonomous decision makers indexed by time t, following Peleg and Yaari (1973) and others.[8] We call the decision maker at time t as self t. As in Pollak (1968) and others, we consider the choices of each self to be the outcome of an intrapersonal game. We follow Barro (1999) in solving the intrapersonal game. One of the advantages of the Barro style solution is that it allows us to compare the results of our model with a wide range of models where agents discount their future utility non-exponentially, including hyperbolic discounting. Hori and Fu-

[7] In a model with a CIA constraint and discrete time, Lucas (1982) makes the same assumption. Walsh (2003, Chapter 3) discusses the roles of this assumption.

[8] See footnote 2.

tagami (2017) discuss other advantages of the Barro style solution (see Section II of Hori and Futagami (2017) for more details).

In the following analyses, we specify the instantaneous utility functions as

$$u_1(c_v) = \log c_v, \quad \text{and} \quad u_2(1 - l_v) = \theta \log(1 - l_v), \tag{7}$$

where θ is a positive constant.

Given future selves' behaviors and the sequence of $\{r_v,\ w_v\}_{v=t}^{\infty}$, self t chooses c_t, l_t, and m_t, which are considered to be constant over the infinitesimal short interval $[t, t+\Delta]$. The objective of self t is approximated as

$$U_t = \int_t^{t+\Delta} z(v,t)dv + \int_{t+\Delta}^{\infty} z(v,t)dv \approx [u_1(c_v) + u_2(1 - l_v)]\,\Delta + \int_{t+\Delta}^{\infty} z(v,t)dv, \tag{8}$$

where $z(v,t) \equiv u_1(c_v)e^{-\rho_c(v-t)} + u_2(1-l_v)e^{-\rho_l(v-t)}$. This approximation comes from taking $e^{-\rho_c(v-t)}$ and $e^{-\rho_l(v-t)}$ as equal to one in the infinitesimally short interval $[t, t+\Delta]$.

Through the choices of c_t, m_t, and l_t, self t influences the behaviors of selves $v(\geq t+\Delta)$ by affecting the asset holdings $a_{t+\Delta}$. To obtain the optimal behavior of self t, we have to first know the effects of c_t, m_t, and l_t on $a_{t+\Delta}$, and then conjecture the policy functions of selves $v(\geq t+\Delta)$.

Budget constraint (3) can be approximated as $a_{t+\Delta} \approx (1+r_t\Delta)a_t + \{w_t l_t - c_t - (r_t+\pi_t)m_t + T_t\}\Delta$ because we can ignore the terms involving Δ^2 and consider r_t, w_t, and π_t as constant in the infinitesimally short time interval $[t, t+\Delta]$. This equation implies that

$$\frac{\partial a_{t+\Delta}}{\partial c_t} = -\Delta, \quad \frac{\partial a_{t+\Delta}}{\partial l_t} = w_t\Delta, \quad \text{and} \quad \frac{\partial a_{t+\Delta}}{\partial m_t} = -(r_t + \pi_t)\Delta. \tag{9}$$

Next, we turn to the policy functions of self $v(\geq t+\Delta)$. Following Barro (1999), we conjecture the choices of self $v(\geq t+\Delta)$ and the path of future consumption as, respectively,

$$1 - l_v = \theta\zeta_v \cdot c_v, \tag{10}$$

$$g_v^c \equiv \frac{\dot{c}_v}{c_v} = r_v - \omega_v. \tag{11}$$

Here, ζ_v and ω_v will be determined later. As in Barro (1999), we conjecture that ζ_v and ω_v do not depend on the level of asset holdings and that ζ_v and ω_v vary over time. In addition, we conjecture that self $v(\geq t+\Delta)$ does not hold more cash than needed for purchasing consumption goods, meaning that (2) holds with equality for all $v(\geq t+\Delta)$. We see that our conjecture turns out to be true.

We integrate (3) using (2), (10), and (11), to obtain $(\mu_{t+\Delta} + \kappa_{t+\Delta})c_{t+\Delta} = a_{t+\Delta} + W_{t+\Delta}$, where $\mu_t \equiv \int_t^{\infty}\{1+\eta(r_v+\pi_v)\}e^{\int_t^v(g_s^c-r_s)ds}dv$, $\kappa_t \equiv \int_t^{\infty}\theta\zeta_v\cdot w_v e^{\int_t^v(g_s^c-r_s)ds}dv$, and $W_t \equiv \int_t^{\infty}\{w_v+T_v\}e^{-\int_t^v r_s ds}dv$. Here, we use our conjecture that (2) holds with equality for all $v(\geq t+\Delta)$. Since we conjecture that both ζ_v and ω_v do not depend on $a_{t+\Delta}$, $a_{t+\Delta}$ has no effects on $\mu_{t+\Delta}$ and $\kappa_{t+\Delta}$. We then have

$$\frac{\partial c_{t+\Delta}}{\partial a_{t+\Delta}} = \frac{1}{\mu_{t+\Delta} + \kappa_{t+\Delta}}. \tag{12}$$

If we use (10) and (11), the objective function of self t, given by (8), can now be rewritten as

$$U_t = [\log c_t + \theta\log(1 - l_t)]\,\Delta + (\log c_{t+\Delta})\,\rho_c^{-1}e^{-\rho_c\Delta} + \theta\,(\log c_{t+\Delta})\,\rho_l^{-1}e^{-\rho_l\Delta} + \Phi_{t+\Delta},$$

where $\Phi_t \equiv \int_t^\infty \left\{\left(\int_t^v g_s^c ds\right) e^{-\rho_c(v-t)} + \theta\left(\log(\theta\zeta_v) + \int_t^v g_s^c ds\right) e^{-\rho_l(v-t)}\right\} dv$. Given the sequence of $\{r_v,\ w_v,\ p_v,\ \pi_v\}_{v=t}^\infty$, self t maximizes this objective function subject to (9), (12), and $m_t \geq \eta c_t$. We set the Lagrangian $\mathcal{L}_t = U_t + \lambda_t(m_t - \eta c_t)$, where λ_t is the Lagrangian multiplier. Note that $a_{t+\Delta}$ has no effect on $\Phi_{t+\Delta}$ because we conjecture that both ζ_v and ω_v do not depend on $a_{t+\Delta}$. Then, the first-order conditions are given by

$$\left(\frac{1}{c_t} - \frac{1}{c_{t+\Delta}} X_\Delta \frac{\partial c_{t+\Delta}}{\partial a_{t+\Delta}}\right)\Delta = \eta\lambda_t, \tag{13}$$

$$\frac{\theta}{1-l_t} = \frac{1}{c_{t+\Delta}} X_\Delta \frac{\partial c_{t+\Delta}}{\partial a_{t+\Delta}} w_t, \tag{14}$$

$$\frac{1}{c_{t+\Delta}} X_\Delta \frac{\partial c_{t+\Delta}}{\partial a_{t+\Delta}} (r_t + \pi_t)\Delta = \lambda_t, \tag{15}$$

where $X_\Delta = \rho_c^{-1} e^{-\rho_c\Delta} + \rho_l^{-1}\theta e^{-\rho_l\Delta}$.

Condition (15) implies that $\lambda_t > 0$, which means that (2) holds with equality for self t. Since this also applies to selves $v(\geq t+\Delta)$, our conjecture that (2) holds with equality for all $v(\geq t+\Delta)$ turns out to be true.

From (13) and (15), we have

$$\frac{1}{c_t} = \frac{X_\Delta}{c_{t+\Delta}} \frac{\partial c_{t+\Delta}}{\partial a_{t+\Delta}} \{1 + \eta(r_t + \pi_t)\}. \tag{16}$$

Using (10), (14), and (16), we obtain

$$\zeta_t = \frac{1 + \eta(r_t + \pi_t)}{w_t}. \tag{17}$$

ζ_t does not depend on a_t. Our conjecture turns out to be true.

From (12) and (16), we have

$$\mu_t + \kappa_t = \{1 + \eta(r_t + \pi_t)\}(\rho_c^{-1} + \theta\rho_l^{-1}), \tag{18}$$

as Δ approaches to zero. From the definitions of μ_t and κ_t as well as (17), we have

$$\dot{\mu}_t = \{r_t - g_t^c\}\mu_t - \{1 + \eta(r_u + \pi_u)\},$$
$$\dot{\kappa}_t = \{r_t - g_t^c\}\kappa_t - \theta\{1 + \eta(r_u + \pi_u)\}.$$

By differentiating both sides of (18) with respect to time and using (11) and the above two equations, we obtain

$$\omega_t = \frac{(1+\theta)\rho_c\rho_l}{\rho_l + \theta\rho_c} + \frac{\eta(\dot{r}_t + \dot{\pi}_t)}{1 + \eta(r_t + \pi_t)}.$$

As conjectured, ω_t does not depend on the level of asset holdings. The behavior of self t can be summarized as

$$\frac{\dot{c}_t}{c_t} = r_t - \tilde{\rho} - \frac{\eta(\dot{r}_t + \dot{\pi}_t)}{1 + \eta(r_t + \pi_t)}, \tag{19}$$

$$1 - l_t = \frac{\theta\{1 + \eta(r_t + \pi_t)\}}{w_t} c_t, \tag{20}$$

where $\tilde{\rho} \equiv (1+\theta)\rho_c\rho_l/(\rho_l+\theta\rho_c)$.

3.1 Observational Equivalence

Consider the following preference with standard exponential discounting:

$$U_t = \int_t^{+\infty} \{u_1(c_v) + u_2(1-l_v)\} e^{-\tilde{\rho}(v-t)} dv. \tag{21}$$

In (21), consumption and leisure are discounted at the same rate. If we maximize the above utility function subject to (2) and (3), we obtain the same equations as (19) and (20). Thus, the behavior of agents endowed with (1) is observationally equivalent to that of agents endowed with (21).

3.2 Inefficiency

In this subsection, we show that the intrapersonal game generates inefficiency. Consider the case where self s believes that self $t(>s)$ will obey the decision made at time s, as in Section 2. Then, (5) holds, and can written as

$$1 - l_t^s = \frac{\theta\{1+\eta(r_t+\pi_t)\}}{w_t} e^{-(\rho_l-\rho_c)(t-s)} c_t^s, \tag{22}$$

under (7). We use superscript s to indicate that c_t^s and l_t^s are optimal for self $s(<t)$. In the intrapersonal game, self $t(>s)$ follows (20), which is not optimal for self s. From the perspective of self s, it is optimal if self $t(>s)$ follows (22). If $\rho_c \neq \rho_l$, (20) does not correspond to (22). Thus, the intrapersonal game generates inefficiency.[9]

Note that the disparity between (20) and (22) comes from the term $e^{-(\rho_l-\rho_c)(t-s)}$. This term, derived from the left-hand side of (5), is the source of time-inconsistency. Thus, time-inconsistency causes intratemporal inefficiency in our model.

To interpret the intratemporal inefficiency, we compare (20) with (22). If $\rho_c > (<)\rho_l$ holds, we have $c_t/(1-l_t) > (<) c_t^s/(1-l_t^s)$ for $t > s$. This equation shows that in the intrapersonal game, self $t(>s)$ consumes more (less) than that self s prefers, given l_t. The intuition is as follows. If $\rho_c > (<)\rho_l$, today's self cares less (more) about consumption of future selves than leisure of future selves. However, future selves care about their own consumption as much as they do about their own leisure. Thus, given l_t, future selves consume more (less) than that today's self prefers.

4 Optimal Monetary Policy

This section examines the effect of monetary policy on inefficiency that is observed in Section 3.2. To characterize an optimal policy, we consider a simple general equilibrium that ensures uniqueness of the equilibrium and simplifies the expression of welfare.

The population size is normalized to one. The production technology is $Y_t = Al_t$, where Y_t is the output level, l_t is the labor input, and A is a positive constant. Through profit maximization, the wage rate w_t becomes equal to A. Because there is no capital, we have $a_t = m_t$. We focus

[9] Hori and Futagami (2017) show that the intrapersonal game generates two types of inefficiencies, intratemporal and intertemporal. The present study focuses on intratemporal inefficiency.

on the steady-state equilibrium, where $\dot{c}_t = \dot{r}_t = \dot{\pi}_t = 0$ holds. Equation (19) implies that $r_t = \tilde{\rho}$. Because $m_t = \eta c_t$ implies that $\dot{c}_t/c_t = \epsilon - \pi_t$, the inflation rate π_t is equal to ϵ. Thus, the nominal interest rate is $\tilde{\rho} + \epsilon$. To ensure $\tilde{\rho} + \epsilon \geq 0$, we assume that $\epsilon \geq -\tilde{\rho}$. From (20), $w_t = A$, $r_t = \tilde{\rho}$, $\pi_t = \epsilon$, and $c_t = Al_t$, we obtain

$$c^* = \frac{A}{1 + \theta\{1 + \eta(\tilde{\rho} + \epsilon)\}} \quad \text{and} \quad 1 - l^* = \frac{\theta\{1 + \eta(\tilde{\rho} + \epsilon)\}}{1 + \theta\{1 + \eta(\tilde{\rho} + \epsilon)\}} = \frac{\theta\{1 + \eta(\tilde{\rho} + \epsilon)\}}{A} c^*. \tag{23}$$

We use asterisks to denote equilibrium variables. Since the above two equations determine c^* and l^* uniquely, the equilibrium is also unique.

By differentiating the two equations of (23) with respect to ϵ, we obtain

$$\frac{\partial c^*}{\partial \epsilon} = -\frac{\theta \eta c^*}{1 + \theta\{1 + \eta(\tilde{\rho} + \epsilon)\}} < 0 \quad \text{and} \quad \frac{\partial(1 - l^*)}{\partial \epsilon} = -\frac{1}{A}\frac{\partial c^*}{\partial \epsilon} > 0. \tag{24}$$

A higher inflation reduces the value of real money and hence depresses consumption. It also decreases the cost of leisure in terms of consumption good, $w/[1+\theta\{1+\eta(\tilde{\rho}+\epsilon)\}]$. Thus, leisure increases with inflation. We also have $\partial c^*/\partial \eta < 0$ and $\partial(1 - l^*)/\partial \eta > 0$. As the financial market develops, the CIA constraint loosens, encouraging consumption and discouraging leisure.

We now discuss our welfare measure. Because of time-inconsistency, different selves of an agent need not agree on the welfare ranking of the same consumption and labor supply sequences. Nevertheless, many studies such as Laibson (1996, 1997) and Laibson et al. (1998) compare welfare from the perspective of all selves. Note that in our model, the utility levels of all selves can be given by $U^* = (\ln c^*)/\rho_c + \{\theta \ln(1 - l^*)\}/\rho_l$. Thus, we can easily evaluate welfare from the perspective of all selves.

To derive the inflation rate ϵ that maximizes U^* (henceforth, ϵ^*), we differentiate U^* with respect to ϵ using (23).

$$\frac{\partial U^*}{\partial \epsilon} = \frac{\theta \eta}{1 + \theta\{1 + \eta(\tilde{\rho} + \epsilon)\}} \left\{ \frac{1}{\rho_l\{1 + \eta(\tilde{\rho} + \epsilon)\}} - \frac{1}{\rho_c} \right\}. \tag{25}$$

By examining the sign of $\partial U^*/\partial \epsilon$, we obtain the next proposition.

Proposition 1

$$\epsilon^* = \begin{cases} -\tilde{\rho}, & \text{if} \quad \rho_c \leq \rho_l, \\ \frac{1}{\eta}\left(\frac{\rho_c}{\rho_l} - 1\right) - \tilde{\rho} (> -\tilde{\rho}), & \text{if} \quad \rho_c > \rho_l, \end{cases}$$

Note that the real interest rate r is equal to $\tilde{\rho}$. When $\epsilon^* = -\tilde{\rho}$, the nominal interest rate is equal to zero. If $\rho_c \leq \rho_l$, a zero nominal interest rate is optimal. Thus, the Friedman rule is optimal. Note that this case includes the standard exponential discounting model where consumption and leisure are discounted at the same rate. When $\rho_c > \rho_l$, ϵ^* is larger than $-\tilde{\rho}$. Here, the Friedman rule is not optimal.

To interpret Proposition 1, we approximate the utility of self s as

$$\begin{aligned} U_s &\approx [u_1(c_s) + u_2(1 - l_s)]\,\Delta + \int_{s+\Delta}^{\infty} \left[u_1(c_t)e^{-\rho_c(t-s)} + u_2(1 - l_t)e^{-\rho_l(t-s)}\right] dt \\ &\approx [u_1(c^*) + u_2(1 - l^*)]\,\Delta + \left\{\frac{1 - \rho_c\Delta}{\rho_c} u_1(c^*) + \frac{1 - \rho_l\Delta}{\rho_l} u_2(1 - l^*)\right\}, \end{aligned} \tag{26}$$

where $\Delta > 0$ is infinitesimally small. The second line uses $e^{-\rho_x \Delta} \approx 1 - \rho_x \Delta$ ($x = c$ or l). The first term in the second line represents the utility of self s from self s's own consumption and leisure at time s. We differentiate the first term with respect to ϵ:

$$\frac{\partial \text{ first term}}{\partial \epsilon} = \left(1 - \frac{1}{1 + \eta(\tilde{\rho} + \epsilon)}\right) \frac{1}{c^*} \frac{\partial c^*}{\partial \epsilon}.$$

This equation shows that the first term is maximized if $\epsilon = -\rho$, because $\partial c^* / \partial \epsilon < 0$ (see (24)). A higher inflation has a negative effect on self s's welfare. This is a standard result. Since self s can control her own behavior, distorting self s's own behavior deteriorates self s's welfare.

The second term in the second line of (26) represents self s's utility from the consumption and leisure of future selves. By differentiating this term with respect to ϵ, we obtain

$$\frac{\partial \text{ second term}}{\partial \epsilon} = \left(\frac{1 - \rho_c \Delta}{\rho_c} - \frac{1 - \rho_l \Delta}{\rho_l} \frac{1}{1 + \eta(\tilde{\rho} + \epsilon)}\right) \frac{1}{c^*} \frac{\partial c^*}{\partial \epsilon}.$$

Because of $\partial c^* / \partial \epsilon < 0$, the second term is maximized if

$$\epsilon = \begin{cases} -\tilde{\rho}, & \text{if} \quad \rho_c \leq \rho_l, \\ \frac{1}{\eta}\left(\frac{\rho_c}{\rho_l} \frac{1 - \rho_l \Delta}{1 - \rho_c \Delta} - 1\right) - \tilde{\rho} (> -\tilde{\rho}), & \text{if} \quad \rho_c > \rho_l, \end{cases}$$

When $\rho_c < \rho_l$ holds, future selves consume less than that today's self prefers, as discussed in Subsection 3.2. An increase in ϵ has a negative effect on the consumption of future selves. Thus, by keeping ϵ as low as possible ($\epsilon = -\rho$), we can maximize the second term. In contrast with this case, when $\rho_c > \rho_l$ holds, future selves consume more than that today's self prefers. Since an increase in ϵ decreases the consumption of future selves, it has a positive effect on the second term.

To maximize U_s, the effects on the first and second terms must be balanced. Thus, we obtain Proposition 1.

We can examine how development of the financial market affects the optimal inflation rate. If $\rho_c > \rho_l$, we have $\partial \epsilon^* / \partial \eta < 0$ from Proposition 1. Thus, we obtain the following corollary.

Corollary
Suppose that $\rho_c > \rho_l$. Then, as the financial market develops, the optimal inflation rate ϵ^* increases.

The intuition of Corollary is simple. When $\rho_c > \rho_l$ holds, future selves consume more than that today's self prefers. Development of the financial market increases the consumption of future selves further. Thus, to maximize the welfare of self s, the consumption of future selves must be depressed through higher inflation. The optimal inflation rate ϵ^* increases with the development of the financial market.

We finally examine the effect of development of the financial market (decreases in η) by keeping ϵ constant at some level. Given $\epsilon (> -\tilde{\rho})$, we differentiate U^* with respect to η by using (23).

$$\frac{\partial U^*}{\partial \eta} = \frac{\theta(\tilde{\rho} + \epsilon)}{1 + \theta\{1 + \eta(\tilde{\rho} + \epsilon)\}} \left\{\frac{1}{\rho_l \{1 + \eta(\tilde{\rho} + \epsilon)\}} - \frac{1}{\rho_c}\right\}. \tag{27}$$

When $\rho_c \leq \rho_l$, $\partial U^* / \partial \eta$ has a negative sign. On the other hand, when $\rho_c > \rho_l$, $\partial U^* / \partial \eta$ has a positive (negative) sign if and only if $\eta < (>) (\rho_c - \rho_l) / \{\rho_l (\tilde{\rho} + \epsilon)\} \equiv \overline{\eta}$. Thus, we obtain the following proposition:

Proposition 2
As the financial market develops (as η decreases),

1. *when $\rho_c \leq \rho_l$ holds, the utility levels of all selves increase, and*
2. *when $\rho_c > \rho_l$ holds, the utility levels of all selves increase if the financial market is less developed ($\eta > \overline{\eta}$) and decrease if the financial market is well developed ($\eta < \overline{\eta}$).*

The intuition of Proposition 2 is similar to that of Proposition 1. As η decreases, the consumption of today's self increases, which has a positive effect on U^*. When $\rho_c > \rho_l$ holds, future selves consume more than that today's self prefers. Thus, a decrease in η has a negative effect on U^* because it increases future selves' consumption. In an economy with a less-developed (well-developed) financial market, the positive (negative) effect dominates the negative (positive) effect. Development of the financial market improves (deteriorates) the utility levels of all selves.

When $\rho_c < \rho_l$ holds, future selves consume less than that today's self prefers. Thus, a decrease in η has a positive effect on U^* because it increases future selves' consumption. Thus, development of the financial market improves the utility levels of all selves.

When $\rho_c = \rho_l$ holds, there is no intratemporal inefficiency. Development of the financial market loosens the CIA constraint, which always has a positive welfare effect.

5 A Discount Rate that Depends on Time Distance

To emphasize the importance of Propositions 1 and 2, this section derives the optimal policy in a model with a discount rate that varies with time distance, as in the model of Barro (1999).

Instead of (1), this section assumes that the representative agent has the following utility function:

$$U_t = \int_t^{+\infty} \{u_1(c_v) + u_2(1 - l_v)\} e^{-(\varrho\cdot(v-t)+\phi(v-t))} dv, \tag{28}$$

where $u_1(c) = \ln c$, $u_2(1 - l) = \theta \ln(1 - l)$, ϱ is a positive constant, and $\phi(v - t)$ is a function of time distance $v-t$. Following Barro (1999), we assume that $\phi(0) = 0$, $\phi'(v-t) \geq 0$, $\phi''(v-t) \leq 0$, and $\lim_{v-t\to\infty} \phi'(v - t) = 0$. In (28), the instantaneous discount rate $\varrho + \phi'(v - t)$ varies with time distance $v-t$. Since we do not specify the functional form of $\phi(v-t)$, the following result applies to a wide range of time-variable discount functions, including hyperbolic discount functions.[10] This generality is an advantage of the Barro-style model.

It is well known that when the discount rate varies with time distance as in (28), the problem of time-inconsistency arises. As in Section 3, we consider the agent as composed of a sequence of autonomous decision makers. The agent is faced with the budget constraint (3) and the CIA constraint (2). By following the same procedure of Section 3, we derive the behavior of self t

[10] A hyperbolic discount function is given by $1/\{1 + f \cdot (v - t))\}$, where $f > 0$ is a constant parameter. Thus, when we define $\phi(v - t)$, which satisfies $e^{-\varrho\cdot(v-t)+\phi(v-t)} = 1/\{1 + f \cdot (v - t))\}$, we obtain a hyperbolic discounting function.

as

$$\frac{\dot{c}_t}{c_t} = r_t - \Omega^{-1} - \frac{\eta(\dot{r}_t + \dot{\pi}_t)}{1 + \eta(r_t + \pi_t)}, \tag{29}$$

$$1 - l_t = \frac{\theta\{1 + \eta(r_t + \pi_t)\}}{w_t} c_t, \tag{30}$$

where $\Omega \equiv \int_0^\infty \exp\{-(\varrho t + \phi(t))\}\, dt > 0$ is a positive constant. See the appendix for the derivations of (29) and (30). Equation (30) is the same as (20). If Ω^{-1} is equal to $\tilde{\rho}(\equiv (1+\theta)\rho_c\rho_l/(\rho_l + \theta\rho_c))$, (29) is exactly the same as (19). The model with a discount rate varying with time distance is observationally equivalent to the non-unitary discount rate model. Furthermore, from the discussion in Subsection 3.1, the behavior of agents with (1) is observationally equivalent to those of agents with (21) or (28).

As in Section 4, we focus on the steady-state equilibrium, where $\dot{c}_t = \dot{r}_t = \dot{\pi}_t = 0$ holds. The production technology is again given by $Y_t = Al_t$. Because of the observational equivalence, the equilibrium consumption level and labor supply are again given by the two equations of (23). In equilibrium, all selves have the same utility level, $U^*_\Omega \equiv [\log(c^*) + \theta \log(1 - l^*)]\Omega$, where c^* and l^* are given by (23).

To derive the optimal ϵ, we differentiate U^*_Ω with respect to ϵ as follows:

$$\frac{\partial U^*_\Omega}{\partial \epsilon} = \frac{\theta\eta}{1 + \theta\{1 + \eta(\tilde{\rho} + \epsilon)\}} \left\{ \frac{1}{1 + \eta(\tilde{\rho} + \epsilon)} - 1 \right\} \Omega. \tag{31}$$

The above equation implies that $\epsilon = -\tilde{\rho}$ is optimal. In spite of time-inconsistency, the Friedman rule is optimal in the model with a discount rate that varies with time distance. This result is sharp contrast with Proposition 1. We next differentiate U^*_Ω with respect to η:

$$\frac{\partial U^*_\Omega}{\partial \eta} = \frac{\theta(\tilde{\rho} + \epsilon)}{1 + \theta\{1 + \eta(\tilde{\rho} + \epsilon)\}} \left\{ \frac{1}{1 + \eta(\tilde{\rho} + \epsilon)} - 1 \right\} \Omega < 0. \tag{32}$$

In contrast to the non-unitary discount rate case, development of the financial market always improves welfare.

Why do we obtain the results that are quite different from Propositions 1 and 2? The answer to this question is simple. If agents are endowed with (28) (or (21)), the same discount function (rate) applies to both consumption and leisure. The inefficiency discussed in Subsection 3.2 does not exist under (28) (or (21)). Thus, the Friedman rule is optimal and development of financial market always improves welfare.

As mentioned above, the behavior of agents with (1) is observationally equivalent to that of agents with (21) or (28). Nevertheless, the three types of preferences derive quite different optimal levels of inflation and financial development.

6 Conclusion

Motivated by recent evidences from psychology and behavioral economics, this study provides a simple model where agents discount the utility from consumption at a rate different from that for the utility of leisure. We show that in our non-unitary discount rate model, the preferences of agents are time-inconsistent. We examine the optimal monetary policy. If a higher discount rate is applied for consumption than for labor, the Friedman rule is no longer optimal. A strictly positive nominal interest rate improves social welfare. In addition, the optimal inflation rate increases as the financial market develops.

We also consider (i) a model with standard exponential discounting and (ii) a model with a discount rate that varies with time distance. Both cases assume that the same discounting applies to both consumption and leisure. The behavior of agents in our non-unitary discount rate model is observationally equivalent to that of agents of the two models. Nevertheless, the optimal monetary policies in the three models are quite different. In the standard exponential discounting model and the model with a discount rate that varies with time distance, the Friedman rule is always optimal.

Appendix

Using the same procedure as in Section 3, we derive (29) and (30). Again, the effects of c_t, l_t, and m_t on $a_{t+\Delta}$ are given by the three equations of (9). As in Section 3, the choices of selves $v(\geq t+\Delta)$ and the path of future consumption are conjectured as

$$1 - l_v = \theta\chi_v \cdot c_v \quad \text{and} \quad \overline{g}_v^c \equiv \frac{\dot{c}_v}{c_v} = r_v - \xi_t.$$

As in Section 3, we conjecture that χ_v and ξ do not depend on the level of asset holdings. The effects of $a_{t+\Delta}$ on $c_{t+\Delta}$ are given by

$$\frac{\partial c_{t+\Delta}}{\partial a_{t+\Delta}} = \frac{1}{\overline{\mu}_{t+\Delta} + \overline{\kappa}_{t+\Delta}},$$

where $\overline{\mu}_t \equiv \int_t^\infty \{1 + \eta(r_v + \pi_v\} e^{\int_t^v \{\overline{g}_s^c - r_s\} ds} dv$ and $\overline{\kappa}_t \equiv \int_t^\infty \theta\chi_v \cdot w_v e^{\int_t^v \{\overline{g}_s^c - r_s\} ds} dv$.

The objective function of self t is given by

$$U_t = [\ln c_t + \theta \ln(1 - l_t)]\,\Delta + (1 + \theta)e^{-(\varrho\cdot\Delta + \phi(\Delta))}\Omega_{t+\Delta} \ln c_{t+\Delta} + \Gamma_{t+\Delta},$$

where $\Omega_t \equiv \int_t^\infty \exp\{-[\varrho\cdot(v-t) + \phi(v-t)]\}\,dv$ and $\Gamma_t \equiv \int_t^\infty \left\{(1+\theta)\int_t^v \overline{g}_s^c ds + \theta \log(\theta\chi_v)\right\} e^{-(\varrho\cdot(v-t)+\phi(v-t))} dv$. Given the sequence of $\{r_v,\ w_v\}_{v=t}^\infty$, self t chooses c_t and l_t so as to maximize this objective function. Note that Ω_t is constant over time because we have $\Omega_t = \int_t^\infty \exp\{-[\varrho\cdot(v-t) + \phi(v-t)]\}\,dv = \int_0^\infty \exp\{-[\varrho\cdot s + \phi(s)]\}\,ds = \Omega_0$. We denote this constant level as Ω.

Using the first-order conditions and limiting Δ to zero, we obtain $\chi_t = [1 + \eta(r_t + \pi_t)]/w_t$ and $\xi = 1/\Omega + \eta(\dot{r}_t + \dot{\pi}_t)/\{1 + \eta(r_t + \pi_t)\}$. Then, we derive (29) and (30).

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