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Abstract

In non-cooperative bargaining models, if negotiators cannot reach an agreement, the bargaining breaks down. Especially, in the standard simultaneous-offers bargaining model, disagreement is supported as an equilibrium outcome. To avoid such disagreement, an arbitrator is often introduced into bargaining. The role of an arbitrator is imposing some agreement as a final bargaining outcome when negotiators cannot reach an agreement by themselves. However, introducing an arbitrator carries a risk that a fair agreement for negotiators is eliminated from equilibrium outcomes if the arbitrator is biased. In our study, to avoid such a risk, we consider introducing a mediator instead of an arbitrator. While an arbitrator imposes an agreement, a mediator can only give advice. We analyze a simultaneous-offers bargaining model with a mediator and obtain the following desirable results. First, disagreement is not supported as an outcome of a stationary subgame perfect equilibrium (SSPE). This result implies that a mediator can resolve conflicts as with an arbitrator. Second, even if a mediator is biased, the fair agreement in the sense of the Nash bargaining solution (NBS) is guaranteed as an SSPE outcome. Therefore, the risk by a biased mediator does not appear. Finally, conversely, if a mediator is fair, the negotiators always reach an agreement with the NBS in SSPE when the discount factor is sufficiently large. That is, the fair mediator facilitates the reaching of a fair agreement.

JEL classification: C72; C73; C78; J52
Keywords: Simultaneous-offers bargaining; Mediator; Bias; Nash bargaining solution; Disagreement

1 Introduction

In non-cooperative bargaining models, if negotiators cannot reach an agreement, the bargaining breaks down (disagreement). Especially, in the standard simultaneous-offers bargaining model, disagreement is supported as an equilibrium outcome.[1] However, in the sense that disagreement is unprofitable, such an outcome is undesirable.

In reality, to avoid such undesirable disagreement, an arbitrator is often introduced into bargaining. The role of an arbitrator is imposing some agreement as a final bargaining outcome when negotiators cannot reach an agreement by themselves. For example, such an arbitrator is used to resolve conflicts in public-sector and to determine the salaries of major league baseball

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[1] The standard simultaneous-offers bargaining is analyzed in Chatterjee and Samuelson (1990) as a generalization of the Nash demand game (Nash (1953)).

players. When an arbitrator is introduced into the bargaining, disagreement vanishes since the arbitrator forces negotiators to reach an agreement.

The bargaining with an arbitrator is analyzed in some papers. For example, Crawford (1979), Yildiz (2011), and Rong (2012) analyze such bargaining. Crawford (1979) analyzes the simultaneous-offers bargaining model, and Yildiz (2011) and Rong (2012) analyze the alternating-offers bargaining models (the model of Rong (2012) is a generalization of the model of Yildiz (2011)). The games of these models proceed as follows. First, negotiators propose their demands simultaneously or alternately. If they can reach an agreement, then the bargaining ends. In contrast, if they cannot reach an agreement by themselves, the game proceeds to the arbitration process and the arbitrator decides a final bargaining outcome.

In these models, since the arbitrators have the authority to decide a final outcome, the equilibrium outcomes strongly depend on what agreement the arbitrator wishes to impose (especially, when the discount factor is sufficiently large). Therefore, a sufficiently "fair" agreement for negotiators which seems to be desirable as a bargaining outcome (e.g. Nash bargaining solution (NBS) (Nash (1950))) can be achieved in equilibrium if and only if the arbitrator is sufficiently fair.

However, unfortunately, in real situations, it is observed that arbitrators are often biased and impose an agreement which seems to be unfair (for example, see Eylon et al. (2000) and Burger and Walters (2005)). Therefore, when an arbitrator is introduced, there is a risk that a fair agreement is eliminated from equilibrium if the arbitrator is biased. Actually, in the models of Crawford (1979) and Rong (2012), such a risk is observed. That is, although an arbitrator is useful to avoid disagreement, introducing an arbitrator carries the risk by a biased arbitrator.

Given these facts, as a way to resolve such a risk, we consider introducing a mediator rather than an arbitrator. Whereas an arbitrator imposes an agreement, a mediator facilitates the reaching of an agreement by negotiators.[2] That is, a mediator can give advice but cannot impose an agreement. In contrast to the bargaining with an arbitrator, negotiators have the right to reject the mediator's advice. In this sense, a mediator has weaker authority than an arbitrator. Such a mediator is also often introduced into bargaining situations, but the role of a mediator in bargaining is not sufficiently analyzed. In this study, we focus on such a mediator.

In our model, the game proceeds as follows. First, the negotiators simultaneously propose their demands. If these demands are compatible, the bargaining ends. If they are incompatible, the bargaining proceeds to the mediation process. In the mediation process, the mediator proposes a plan of an agreement. If both negotiators accept it, the bargaining ends. If some negotiator rejects it, the negotiators propose their demands again and the above process is repeated.

In this study, we show that the following desirable results appear by introducing a mediator, where the NBS plays an important role as a fair agreement. First, we find that, although a mediator cannot impose an agreement, disagreement is not supported as an outcome of stationary subgame perfect equilibrium (SSPE). This result implies that a mediator can resolve conflicts as with an arbitrator. Second, in contrast to a model with an arbitrator, even if a mediator is biased, the fair agreement in the sense of the NBS is guaranteed as an SSPE agreement. Therefore, the risk by a biased mediator does not appear. Additionally, we show that an agreement having such a property is only the NBS. Finally, we show that, conversely, if a mediator is fair in the sense that she wishes to achieve the NBS, the negotiators always reach an agreement with the NBS in SSPEs when the discount factor is sufficiently large. That is, we find that a fair mediator facilitates the reaching of a fair agreement. These are our main results.

In addition to these desirable results, introducing a mediator instead of an arbitrator has another advantage. In reality, calling an arbitrator into bargaining often requires considerable

[2] This definition is by Muthoo (1999).

effort since it may need legal processes. For example, the arbitration in labor dispute often needs it. Also, in reality, there are some bargaining situations where it is difficult to introduce an arbitrator due to negotiators' strong power. For example, in conflicts between nations, since nations have strong power, they may deviate from imposed decision forcibly after negotiation ends. Therefore, to impose an agreement surely, the introduced arbitrator needs to have sufficiently strong prower, but it is difficult to find such an arbitrator. In contrast to these difficulties, since a mediator is merely an adviser, introducing it is easier than an arbitrator. That is, a mediator can resolve conflicts as with an arbitrator, but introducing it does not require much effort. This is another advantage of introducing a mediator.

In the remaining of this section, we introduce other related literatures. Although the role of a mediator in bargaining situations is not sufficiently analyzed, there are a few papers which analyze the bargaining with a mediator (e.g. Wilson (2001) and Jarque et al. (2003)). In most of these papers, a mediator is introduced as a system of the game. That is, a mediator does not make decision and does not have utility. However, since a mediator may have bias, it is natural to consider a mediator as a player of the game rather than a system. Therefore, in our model, we introduce a mediator as a player.

Camiña and Porteiro (2009) introduce a mediator as a player and analyze peace negotiations. In their alternating-offers bargaining model, the roles of the mediator are deciding which negotiator proposes first or deciding whether to submit an offer received from a negotiator to the other negotiator. Therefore, their mediator does not give advice about what agreement negotiators should reach. In contrast to their model, we consider the model where the mediator can propose a plan of an agreement to negotiators.

Manzini and Mariotti (2001) and Manzini and Mariotti (2004) analyze alternating-offers bargaining models with an arbitrator. In these models, the arbitrator imposes an agreement if and only if both negotiators consent to proceed to the arbitration process. Our model and these models are quite different, but they are similar in the sense that the consent from both negotiators is necessary before the mediator's proposal or the arbitrator's decision is implemented.

Manzini and Ponsatí (2005), Manzini and Ponsati (2006), and Ponsatí (2004) analyze the bargaining with a stakeholder. A stakeholder is a third party who is interested in the resolution of the conflict and receives benefits when negotiators reach an agreement. For example, in conflicts in public-sector, the government can be considered as a stakeholder. In this situation, since the government wishes to improve social welfare, it makes effort to resolve the conflicts for its benefit. In our model, the mediator can also be considered as such a stakeholder. The most different point between the above existing literatures and our model is that, whereas the stakeholders in the above literatures are not interested in what agreement the negotiators reach, the mediator of our model is interested in it. At this point, our model can be applied to many bargaining situations such as the bargaining in public sector where the government is interested in how negotiators reach an agreement.

This paper is organized as follows. In section 2, we define a simultaneous-offers bargaining model with a mediator. In section 3, we derive SSPEs of our model and analyze properties of the NBS as an SSPE agreement. In section 4, we compare our model with a model without a mediator and a model with an arbitrator. In this section, we analyze how the mediator affects the bargaining outcomes. In section 5, we conclude our study.

2 The model

We consider a bargaining model with three players, negotiators 1, 2, and the mediator. Let $S \subseteq \mathbb{R}^2_+$ be the feasible utility space for the negotiators. We assume that $d = (0, 0)$ is an element

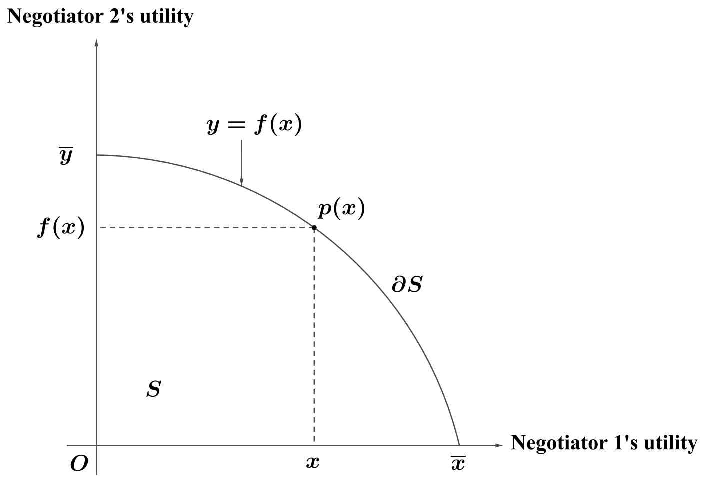

Figure 1: Feasible utility space S for the negotiators

of S, and there exists some $(x, y) \in S$ such that $(x, y) \gg d$.[3] Furthermore, we impose some assumptions on the set S as the same as existing literatures. That is, we assume that the set S is convex, compact, and strictly comprehensive.[4] Also, we define $u : S \to \mathbb{R}_+$ as the mediator's utility function (the mediator is interested in what agreement the negotiators reach). Here, we assume $u(x, y) \geq 0$ for all $(x, y) \in S$.

Additionally, we define $\overline{x} = \max\{x \mid (x, y) \in S\}$, $\overline{y} = \max\{y \mid (x, y) \in S\}$, and the function $f : [0, \overline{x}] \to [0, \overline{y}]$ as $f(x) = \max\{y \mid (x, y) \in S\}$. Since S is compact, these definitions are well-defined. By the assumptions of S, we can confirm that the function f is concave, strictly decreasing, and continuous. Then, $f(0) = \overline{y}$, $f(\overline{x}) = 0$, and f has the inverse function f^{-1} : $[0, \overline{y}] \to [0, \overline{x}]$ represented by $f^{-1}(y) = \max\{x \mid (x, y) \in S\}$ (f^{-1} is also concave, strictly decreasing, and continuous). Also, we define $p(x) = (x, f(x))$ for $x \in [0, \overline{x}]$. Then, the Pareto frontier of S can be represented as $\partial S = \{p(x) \mid x \in [0, \overline{x}]\}$. These are depicted in Figure 1.

Now, we describe the game. The game starts from period 1 and proceeds as follows. Let $\delta \in (0, 1)$ be the common discount factor. At odd period t, the negotiators 1 and 2 simultaneously propose their demands $x \in [0, \overline{x}]$ and $y \in [0, \overline{y}]$, respectively. If $(x, y) \in S$, that is, if the negotiators' demands are compatible, the game ends and negotiators 1, 2, and the mediator receive $\delta^{t-1}x$, $\delta^{t-1}y$, and $\delta^{t-1}u(x, y)$, respectively. If $(x, y) \notin S$, that is, if the negotiators' demands are incompatible, the game proceeds to the next period $t+1$. At even period $t+1$, the mediator proposes some $p(z) \in \partial S$ such that $z \in [f^{-1}(y), x]$ or chooses pass. Now, notice that, when $z \in [f^{-1}(y), x]$, $z \leq x$ and $f(x) \leq y$ hold (see Figure 2). That is, when the mediator gives advice, she recommends the negotiators to concede.

If the mediator chooses pass, then the game proceeds to period $t+2$. If the mediator proposes some $p(z)$, then the negotiators simultaneously decide whether to accept the mediator's proposal or reject it. If both negotiators accept it, the game ends and negotiators 1, 2, and the mediator receive $\delta^t z$, $\delta^t f(z)$, and $\delta^t u(p(z))$, respectively. If some negotiator rejects it, the game proceeds

[3] $(x, y) \gg (x', y')$ denotes $x > x'$ and $y > y'$.

[4] $(x, y) \geqslant (x', y')$ denotes $x \geq x'$ and $y \geq y'$. The set S is *comprehensive* if $(x'', y'') \geqslant (x', y') \geqslant (0, 0)$ and $(x'', y'') \in S$ imply $(x', y') \in S$. The set S is *strictly comprehensive* if S is comprehensive and, for all $(x, y) \in S$ such that $(x', y') \geqslant (x, y)$ and $(x', y') \neq (x, y)$ for some $(x', y') \in S$, there exists some $(x'', y'') \in S$ such that $(x'', y'') \gg (x, y)$.

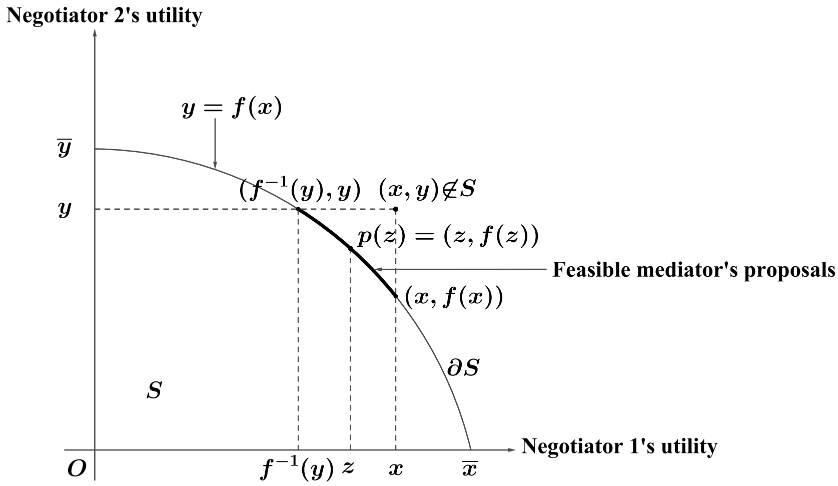

Figure 2: Feasible mediator's proposals

to the next period $t+2$. At period $t+2$ or later, the process at period t is repeated at every odd period and the process at period $t+1$ is repeated at every even period. This game continues until some agreement is reached. If the negotiation continues permanently in some strategy profile, then negotiator 1, 2, and the mediator receive payoffs of zero. The game tree is depicted in Figure 3.

In this study, we suppose that the mediator's utility function is single-peaked on ∂S. That is, we suppose that the mediator considers some agreement on ∂S as the ideal agreement of the bargaining, and suppose that the mediator's utility decreases as the distance from her ideal agreement increases. Formally, we impose the following assumption on the mediator's utility function.

Assumption 1. *With respect to the mediator's utility function u, there exists some $\alpha \in [0, \overline{x}]$ such that,*

1. *for $x, x' \in [0, \overline{x}]$ such that $x < x' \leq \alpha$, $u(p(x)) < u(p(x'))$ and*
2. *for $x, x' \in [0, \overline{x}]$ such that $x > x' \geq \alpha$, $u(p(x)) < u(p(x'))$.*

In this assumption, the mediator's ideal agreement is $p(\alpha) \in \partial S$. The mediator favors negotiator 1 when α is close to $\overline{x}$ and favors negotiator 2 when α is close to zero.

As a solution concept of the above model, we use a stationary subgame perfect equilibrium (SSPE), that is, use a subgame perfect equilibrium (SPE) in which,

1. each negotiator's demand at every odd period is always the same value,
2. the mediator's proposal at every even period depends only on the negotiators' demands at the previous odd period, and
3. for each negotiator, whether she accepts the mediator's proposal or rejects it depends only on the negotiators' demands at the previous odd period and the mediator's proposal at the current period.

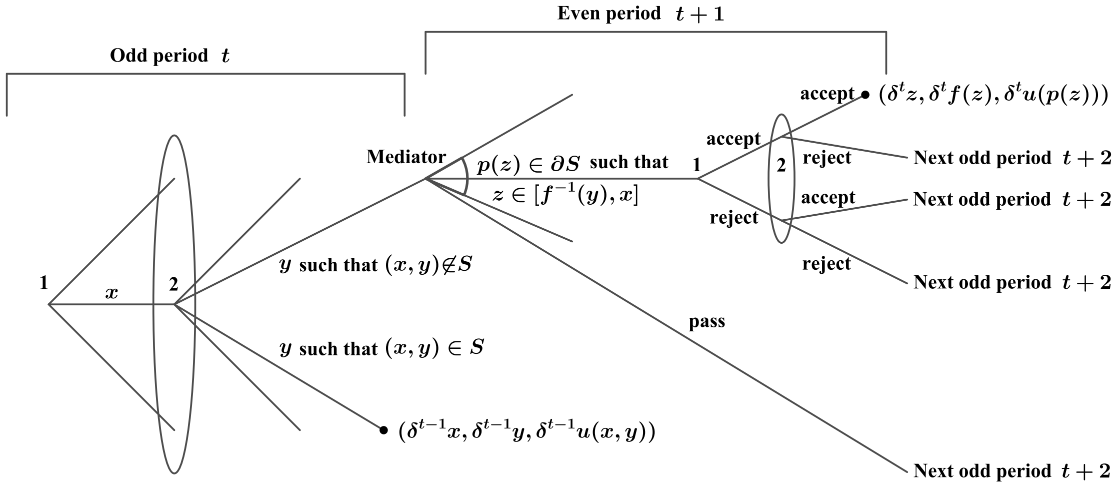

Figure 3: Game tree

Then, we assume that, when some negotiator responses to the mediator's proposal, she accepts it if the mediator's proposal is not less profitable than rejecting it. Thus, if accepting the mediator's proposal and rejecting it are indifferent, the negotiator accepts the mediator's proposal.

3 SSPE outcomes

In this section, we derive SSPE outcomes of our model. Before proceeding to the analysis of SSPE, we prepare additional notation. When the curve $y = f(x)$ is scaled down by δ in the direction of y, we obtain $y = \delta f(x)$. In contrast, when $y = f(x)$ is scaled down by δ in the direction of x, we obtain $y = f(\frac{x}{\delta})$. Let $x^R(\delta)$ be the solution of $f(x) = \delta f(\delta x)$. Then, the unique intersection of the curves $y = \delta f(x)$ and $y = f(\frac{x}{\delta})$ is $(\delta x^R(\delta), f(x^R(\delta)))$. Now, notice that $p(x^R(\delta)) = (x^R(\delta), f(x^R(\delta)))$ and $p(\delta x^R(\delta)) = (\delta x^R(\delta), f(\delta x^R(\delta)))$ are the negotiators' offers proposed in the SPE of the Rubinstein's alternating-offers bargaining (Rubinstein (1982)) with the utility space S. Also, notice that

$$\delta f(x) < f(\frac{x}{\delta}) \text{ when } x \in [0, \delta x^R(\delta)), \tag{1}$$

$$\delta f(x) > f(\frac{x}{\delta}) \text{ when } x \in (\delta x^R(\delta), \delta \overline{x}], \tag{2}$$

$$\delta f^{-1}(y) < f^{-1}(\frac{y}{\delta}) \text{ when } y \in [0, f(x^R(\delta))), \text{ and} \tag{3}$$

$$\delta f^{-1}(y) > f^{-1}(\frac{y}{\delta}) \text{ when } y \in (f(x^R(\delta)), \delta \overline{y}]. \tag{4}$$

The equation $\delta f(x) = f(\frac{x}{\delta})$ holds if and only if $x = \delta x^R(\delta)$ and the equation $\delta f^{-1}(y) = f^{-1}(\frac{y}{\delta})$ holds if and only if $y = f(x^R(\delta))$. These are depicted in Figure 4. In this paper, we use x^R instead of $x^R(\delta)$ when we fix the value of δ.

Bargaining outcomes of our model can be divided into the following three cases.

1. The negotiators reach an agreement with some $(x, y) \in S$ at some odd period t by themselves. This outcome is denoted by $((x, y), t)$ where t is odd.

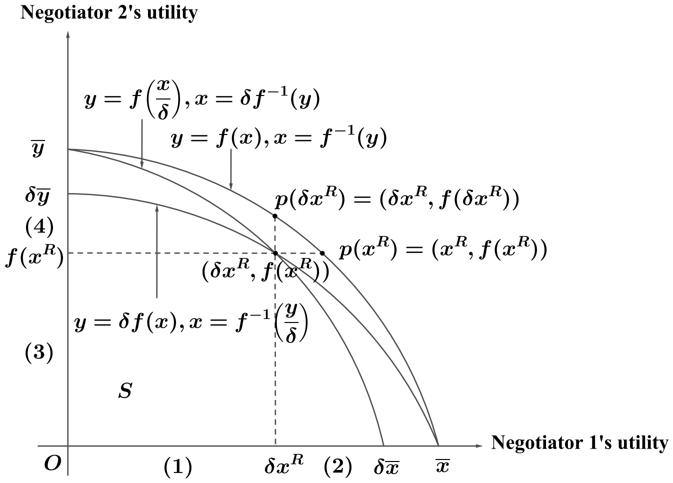

Figure 4: $y = \delta f(x)$ and $y = f(\frac{x}{\delta})$

2. The mediator's proposal $p(z) \in \partial S$ is accepted at some even period t. This outcome is denoted by $(p(z), t)$ where t is even.

3. The negotiators never reach an agreement (disagreement).

In the following subsections, we sequentially analyze each case and derive SSPE outcomes. Also, we analyze properties of an agreement with the NBS as an SSPE agreement. In this study, we use the one-shot deviation principle to derive SSPEs. That is, we use the fact that a stationary strategy profile σ is an SSPE if and only if there is no player who can become better off by deviating from σ for just one period (for example, see Fudenberg and Tirole (1991)).

3.1 SSPE agreement at odd periods

In this subsection, we analyze SSPE outcomes such that the negotiators reach an agreement at odd periods. By the definition of SSPE, in such SSPE outcomes, the negotiators reach an agreement at period 1.

First of all, we prove that, for all $(x, y) \in S \backslash \partial S$, the outcome $((x, y), 1)$ is not supported as an SSPE outcome. That is, if the negotiators reach an agreement by themselves in some SSPE, they always reach an agreement on ∂S.

Lemma 1. *For all $(x, y) \in S \backslash \partial S$, the outcome $((x, y), 1)$ is not supported as an SSPE outcome.*

Proof. Suppose that there exists an SSPE such that the negotiators reach an agreement $(x, y) \in S \backslash \partial S$ at period 1. Then, by the assumption on S, negotiators 1 and 2 can improve their payoffs by deviating from the SSPE and proposing $f^{-1}(y)(> x)$ and $f(x)(> y)$, respectively. This is a contradiction. Thus, $((x, y), 1)$ is not supported as an SSPE outcome. $\square$

Therefore, for deriving SSPE agreement at odd periods, it is sufficient to focus on an agreement on ∂S. Next, we describe the mediator's strategy in the SSPEs where negotiators 1 and 2 demand some x and $f(x)$ at odd periods, respectively.

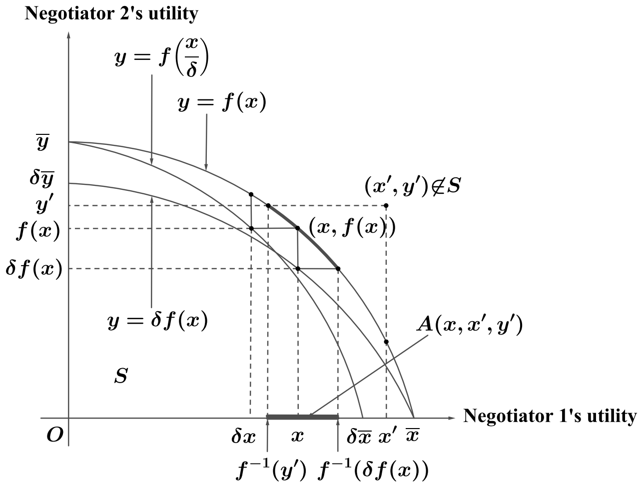

Figure 5: Example of $A(x, x', y')$

Lemma 2. *Suppose that, in some SSPE σ, negotiators 1 and 2 demand x and $f(x)$ at odd periods, respectively, and reach an agreement with $p(x) \in \partial S$. Then, in the path after negotiators 1 and 2 demand x' and y', respectively (where $(x', y') \notin S$), the mediator chooses the following action at even periods under σ. Now, we define $A(x, x', y') = [f^{-1}(y'), x'] \cap [\delta x, f^{-1}(\delta f(x))]$ (see Figure 5).*

1. *When $\alpha \in A(x, x', y')$, the mediator proposes $p(\alpha) \in \partial S$ (it is accepted by both negotiators).*
2. *When $A(x, x', y') \neq \emptyset$, $\alpha < \min A(x, x', y')$, and $\min A(x, x', y') \leq x$, the mediator proposes $p(\min A(x, x', y')) \in \partial S$ (it is accepted by both negotiators).*
3. *When $A(x, x', y') \neq \emptyset$, $\alpha < \min A(x, x', y')$, and $\min A(x, x', y') > x$, the mediator proposes $p(\min A(x, x', y')) \in \partial S$ if $u(p(\min A(x, x', y'))) > \delta u(p(x))$ (it is accepted by both negotiators), and chooses pass (or offers some proposal rejected by some negotiator) if $u(p(\min A(x, x', y'))) < \delta u(p(x))$. If $u(p(\min A(x, x', y'))) = \delta u(p(x))$, the mediator proposes $p(\min A(x, x', y'))$ or chooses pass (or offers some proposal rejected by some negotiator).*
4. *When $A(x, x', y') \neq \emptyset$, $\alpha > \max A(x, x', y')$, and $\max A(x, x', y') \geq x$, the mediator proposes $p(\max A(x, x', y')) \in \partial S$ (it is accepted by both negotiators).*
5. *When $A(x, x', y') \neq \emptyset$, $\alpha > \max A(x, x', y')$, and $\max A(x, x', y') < x$, the mediator proposes $p(\max A(x, x', y')) \in \partial S$ if $u(p(\max A(x, x', y'))) > \delta u(p(x))$ (it is accepted by both negotiators), and chooses pass (or offers some proposal rejected by some negotiator) if $u(p(\max A(x, x', y'))) < \delta u(p(x))$. If $u(p(\max A(x, x', y'))) = \delta u(p(x))$, the mediator proposes $p(\max A(x, x', y'))$ or chooses pass (or offers some proposal rejected by some negotiator).*
6. *When $A(x, x', y') = \emptyset$, the mediator proposes some $p(z) \in \partial S$ satisfying $z \in [f^{-1}(y'), x']$ (it is rejected by some negotiator) or chooses pass.*

Proof. Without loss of generality, in the following proofs, we consider that the negotiators propose their demands at period t and the mediator proposes at period $t+1$. First of all, notice the following facts. By rejecting the mediator's proposal, negotiators 1 and 2 obtain payoffs $\delta^{t+1}x$ and $\delta^{t+1}f(x)$ at period $t+2$, respectively, under σ. Therefore, under σ, in the path after x' and y' are demanded by the negotiators, the mediator's proposal $p(z) \in \partial S$ is accepted by both negotiators if and only if $z \in A(x, x', y')$. Also, notice that, under σ, the mediator obtains $\delta^{t+1}u(p(x))$ at period $t+2$ by choosing pass (or by offering some proposal rejected by some negotiator) at period $t+1$. By the above facts, we prove each case.

1. When $\alpha \in A(x, x', y')$, the mediator's ideal agreement $p(\alpha)$ is accepted by both negotiators. Then, the mediator can obtain $\delta^t u(p(\alpha))$ ($\geq \delta^t u(p(x)) > \delta^{t+1}u(p(x))$ by Assumption 1). Therefore, proposing $p(\alpha) \in \partial S$ is a best response to σ.

2. When $A(x, x', y') \neq \emptyset$, $\alpha < \min A(x, x', y')$, and $\min A(x, x', y') \leq x$, the most profitable proposal for the mediator in $p(A(x, x', y'))$ is $p(\min A(x, x', y')))$.[5] Then, she obtains $\delta^t u(p(\min A(x, x', y')))$ ($\geq \delta^t u(p(x)) > \delta^{t+1}u(p(x))$ by Assumption 1). Therefore, proposing $p(\min A(x, x', y'))) \in \partial S$ is a best response to σ.

3. When $A(x, x', y') \neq \emptyset$, $\alpha < \min A(x, x', y')$, and $\min A(x, x', y') > x$, the most profitable proposal for the mediator in $p(A(x, x', y'))$ is $p(\min A(x, x', y')))$. Then, she obtains $\delta^t u(p(\min A(x, x', y')))$. Therefore, under σ, the mediator proposes $p(\min A(x, x', y')) \in \partial S$ if $u(p(\min A(x, x', y'))) > \delta u(p(x))$, and chooses pass (or offers some proposal rejected by some negotiator) if $u(p(\min A(x, x', y'))) < \delta u(p(x))$. If $u(p(\min A(x, x', y'))) = \delta u(p(x))$, the mediator proposes $p(\min A(x, x', y'))$ or chooses pass (or offers some proposal rejected by some negotiator).

4. Since the proof of this case is analogous to the case 2, we omit it.

5. Since the proof of this case is analogous to the case 3, we omit it.

6. When $A(x, x', y') = \emptyset$, the mediator's proposal $p(z)$ such that $z \in [f^{-1}(y'), x']$ is rejected by some negotiator under σ. Then, by proposing some $p(z)$ or choosing pass, the mediator obtains $\delta^{t+1}u(p(x))$. Therefore, proposing some $p(z)$ satisfying $z \in [f^{-1}(y'), x']$ and choosing pass are best responses to σ.

□

By using Lemma 2, we derive all SSPE agreements at period 1. The analysis is divided into three cases, that is, when $\alpha \in [0, \delta x^R)$, when $\alpha \in [\delta x^R, x^R]$, and when $\alpha \in (x^R, \overline{x}]$. First, when $\alpha \in [0, \delta x^R)$, we obtain the following result.

Lemma 3. *When $\alpha \in [0, \delta x^R)$, the outcome $(p(x), 1)$ is supported as an SSPE outcome if and only if $x \in [\delta\alpha, x^R]$ (see Figure 6). In the SSPE, negotiators 1 and 2 demand x and $f(x)$, respectively, and the mediator follows the strategy described in Lemma 2.*

Proof. By Lemma 1, it is sufficient to consider the case where the negotiators reach an agreement on ∂S. We sequentially analyze five cases with respect to the value of x. Without loss of generality, we consider that the negotiators propose their demands at period t and the mediator proposes at period $t+1$.

[5] For a function p and a set A, we define $p(A) = \{p(a) \mid a \in A\}$.

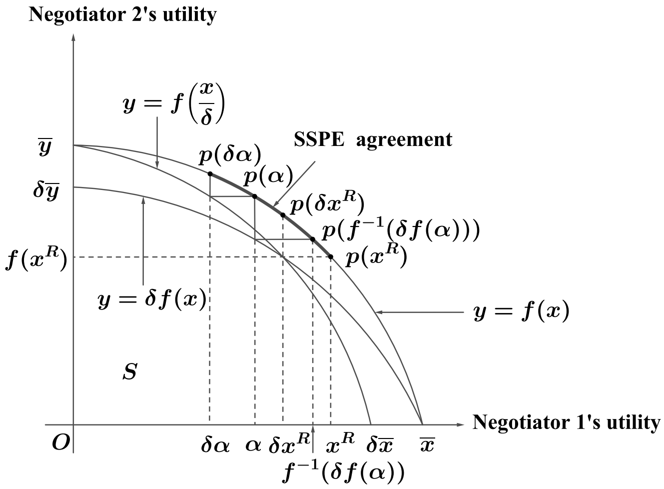

Figure 6: SSPE agreement at period 1 when $\alpha \in [0, \delta x^R)$

1. Suppose that, in some SSPE σ, negotiators 1 and 2 demand x and $f(x)$ such that $x \in [0, \delta\alpha)$, respectively. Consider the case that negotiator 1 deviates from σ and demands $f^{-1}(\delta f(x))$ (notice that $f^{-1}(\delta f(x)) > x$ by the facts that $f(x) > \delta f(x)$ and f^{-1} is strictly decreasing). Then, the game proceeds to the next period and the mediator proposes some $p(z)$ such that $z \in [x, f^{-1}(\delta f(x))]$ or chooses pass. Now, $A(x, f^{-1}(\delta f(x)), f(x)) = [x, f^{-1}(\delta f(x))] \cap [\delta x, f^{-1}(\delta f(x))] = [x, f^{-1}(\delta f(x))]$ $(\neq \emptyset)$.

   If $\alpha \leq f^{-1}(\delta f(x))$, since $x < \delta\alpha < \alpha$, the mediator proposes $p(\alpha)$ by the case 1 of Lemma 2, and negotiator 1 obtains $\delta^t\alpha$ $(> \delta^{t-1}x)$. If $\alpha > f^{-1}(\delta f(x))$, the mediator proposes $p(f^{-1}(\delta f(x)))$ by the case 4 of Lemma 2, and negotiator 1 obtains $\delta^t f^{-1}(\delta f(x))$. Now, since $x < \delta\alpha < \delta x^R$ and $\delta f(\delta x^R) = f(x^R)$, we find $\delta f(x) > \delta f(\delta x^R) = f(x^R)$. Thus, by the inequality (4), we obtain $\delta f^{-1}(\delta f(x)) > x$, that is, $\delta^t f^{-1}(\delta f(x)) > \delta^{t-1}x$.

   Therefore, negotiator 1 can improve her payoff by deviating from σ and demanding $f^{-1}(\delta f(x))$. This is a contradiction. Thus, the outcome $(p(x), 1)$ such that $x \in [0, \delta\alpha)$ is not supported as an SSPE outcome.

2. Suppose that, in some SSPE σ, negotiators 1 and 2 demand x and $f(x)$ such that $x \in [\delta\alpha, \alpha)$, respectively. Then, the mediator's proposals in σ are described in Lemma 2.

   Suppose that negotiator 1 deviates from σ and demands x^* such that $x^* < x$. Then, negotiator 1 obtains $\delta^{t-1}x^*$ $(< \delta^{t-1}x)$. Therefore, she cannot improve her payoff by demanding x^* $(< x)$. Also, suppose that negotiator 2 deviates from σ and demands y^* such that $y^* < f(x)$. Then, negotiator 2 obtains $\delta^{t-1}y^*$ $(< \delta^{t-1}f(x))$. Therefore, she also cannot improve her payoffs by demanding y^* $(< f(x))$.

   Next, suppose that negotiator 1 deviates from σ and demands x^{**} such that $x^{**} > x$. Then, the game proceeds to the next period and the mediator proposes some $p(z)$ such that $z \in [x, x^{**}]$ or chooses pass. Since $x < f^{-1}(\delta f(x))$, $A(x, x^{**}, f(x))$ can be transformed as $A(x, x^{**}, f(x)) = [x, x^{**}] \cap [\delta x, f^{-1}(\delta f(x))] = [x, \min\{x^{**}, f^{-1}(\delta f(x))\}]$ $(\neq \emptyset)$. If $\alpha \leq \min\{x^{**}, f^{-1}(\delta f(x))\}$, since $x < \alpha$, the mediator proposes $p(\alpha)$ by the case 1 of Lemma 2, and negotiator 1 obtains $\delta^t\alpha$ $(\leq \delta^{t-1}x)$. If $\alpha > \min\{x^{**}, f^{-1}(\delta f(x))\}$, the mediator proposes $p(\min\{x^{**}, f^{-1}(\delta f(x))\})$ by the case 4 of Lemma 2, and negotiator 1 obtains $\delta^t \min\{x^{**}, f^{-1}(\delta f(x))\}$ $(< \delta^t \alpha \leq \delta^{t-1} x)$. Therefore, negotiator 1 cannot improve her payoff by demanding x^{**} $(> x)$.

Also, suppose that negotiator 2 deviates from σ and demands y^{**} such that $y^{**} > f(x)$. Then, the game proceeds to the next period and the mediator proposes some $p(z)$ such that $z \in [f^{-1}(y^{**}), x]$ or chooses pass. Now, since $x < f^{-1}(\delta f(x))$, $A(x, x, y^{**}) = [f^{-1}(y^{**}), x] \cap [\delta x, f^{-1}(\delta f(x))] = [\max\{f^{-1}(y^{**}), \delta x\}, x]$ $(\neq \emptyset)$. Therefore, since $x < \alpha$, the mediator proposes $p(x)$ by the case 4 of Lemma 2, and negotiator 2 obtains $\delta^t f(x)$ $(< \delta^{t-1} f(x))$. Thus, negotiator 2 cannot improve her payoff by demanding y^{**} $(> f(x))$.

Consequently, we can find that the SSPE σ is consistent. Therefore, the outcome $(p(x), 1)$ such that $x \in [\delta\alpha, \alpha)$ is supported as an SSPE outcome.

3. Suppose that, in some SSPE σ, negotiators 1 and 2 demand x and $f(x)$ such that $x \in [\alpha, f^{-1}(\delta f(\alpha))]$, respectively. Then, the mediator's proposals in σ are described in Lemma 2. By the same way as the case 2, we can find that negotiator 1 cannot improve her payoff by deviating from σ and demanding x^* $(< x)$. Also, negotiator 2 cannot improve her payoff by demanding y^* $(< f(x))$.

   Suppose that negotiator 1 deviates from σ and demands x^{**} such that $x^{**} > x$. Then, the game proceeds to the next period and the mediator proposes some $p(z)$ such that $z \in [x, x^{**}]$ or chooses pass. Since $x < f^{-1}(\delta f(x))$, $A(x, x^{**}, f(x))$ can be transformed as $A(x, x^{**}, f(x)) = [x, x^{**}] \cap [\delta x, f^{-1}(\delta f(x))] = [x, \min\{x^{**}, f^{-1}(\delta f(x))\}]$ $(\neq \emptyset)$. Then, since $x \geq \alpha$, the mediator proposes $p(x)$ by the case 2 (or 1) of Lemma 2, and negotiator 1 obtains $\delta^t x$ $(< \delta^{t-1} x)$. Thus, negotiator 1 cannot improve her payoff by demanding x^{**} $(> x)$.

   Also, suppose that negotiator 2 deviates from σ and demands y^{**} such that $y^{**} > f(x)$. Then, the game proceeds to the next period and the mediator proposes some $p(z)$ such that $z \in [f^{-1}(y^{**}), x]$ or chooses pass. Now, $A(x, x, y^{**}) = [f^{-1}(y^{**}), x] \cap [\delta x, f^{-1}(\delta f(x))] = [\max\{f^{-1}(y^{**}), \delta x\}, x]$ $(\neq \emptyset)$.

   If $\alpha \geq \max\{f^{-1}(y^{**}), \delta x\}$, since $\alpha \leq x$, the mediator proposes $p(\alpha)$ by the case 1 of Lemma 2, and negotiator 2 obtains $\delta^t f(\alpha)$. Since $x \leq f^{-1}(\delta f(\alpha))$, we obtain $\delta f(\alpha) \leq f(x)$, that is, $\delta^t f(\alpha) \leq \delta^{t-1} f(x)$. If $\alpha < \max\{f^{-1}(y^{**}), \delta x\}$, the mediator proposes $p(\max\{f^{-1}(y^{**}), \delta x\})$ by the case 2 of Lemma 2. Then, negotiator 2 obtains $\delta^t f(\max\{f^{-1}(y^{**}), \delta x\})$ $(< \delta^t f(\alpha))$. Since $x \leq f^{-1}(\delta f(\alpha))$, we obtain $\delta f(\alpha) \leq f(x)$. Therefore, $\delta^t f(\max\{f^{-1}(y^{**}), \delta x\}) < \delta^t f(\alpha) \leq \delta^{t-1} f(x)$ holds. By the above discussion, negotiator 2 cannot improve her payoff by demanding y^{**} $(> f(x))$.

   Consequently, we can find that the SSPE σ is consistent. Therefore, the outcome $(p(x), 1)$ such that $x \in [\alpha, f^{-1}(\delta f(\alpha))]$ is supported as an SSPE outcome.

4. Suppose that in some SSPE σ, negotiators 1 and 2 demand x and $f(x)$ such that $x \in (f^{-1}(\delta f(\alpha)), x^R]$, respectively. Then, the mediator's proposals in σ are described in Lemma 2. By the same way as the case 2, we can find that negotiator 1 cannot improve her payoff by deviating from σ and demanding x^* $(< x)$. Also, negotiator 2 cannot improve her payoff by demanding y^* $(< f(x))$.

   Suppose that negotiator 1 deviates from σ and demands x^{**} such that $x^{**} > x$. Then, the game proceeds to the next period and the mediator proposes some $p(z)$ such that $z \in [x, x^{**}]$ or chooses pass. Now, $A(x, x^{**}, f(x))$ can be transformed as $A(x, x^{**}, f(x)) = [x, x^{**}] \cap [\delta x, f^{-1}(\delta f(x))] = [x, \min\{x^{**}, f^{-1}(\delta f(x))\}]$ $(\neq \emptyset)$. Since $\alpha < f^{-1}(\delta f(\alpha))$, we obtain $x > f^{-1}(\delta f(\alpha)) > \alpha$. Therefore, the mediator proposes $p(x)$ by the case 2 of Lemma

2. Then, negotiator 1 obtains $\delta^t x$ ($< \delta^{t-1}x$). Thus, negotiator 1 cannot improve her payoff by demanding x^{**} ($> x$).

Next, suppose that negotiator 2 deviates from σ and demands y^{**} such that $y^{**} > f(x)$. Then, the game proceeds to the next period and the mediator proposes some $p(z)$ such that $z \in [f^{-1}(y^{**}), x]$ or chooses pass. Now, $A(x, x, y^{**}) = [f^{-1}(y^{**}), x] \cap [\delta x, f^{-1}(\delta f(x))] = [\max\{f^{-1}(y^{**}), \delta x\}, x]$ ($\neq \emptyset$).

Since $\alpha < \delta x^R$, we obtain $\delta f(\alpha) > \delta f(\delta x^R) = f(x^R)$. By the inequality (4), $\delta f^{-1}(\delta f(\alpha)) > \alpha$ holds. Then, since $x > f^{-1}(\delta f(\alpha))$, we obtain $\delta x > \delta f^{-1}(\delta f(\alpha)) > \alpha$. Therefore, since $\alpha < \delta x \leq \max\{f^{-1}(y^{**}), \delta x\}$, the mediator proposes $p(\max\{f^{-1}(y^{**}), \delta x\})$ by the case 2 of Lemma 2, and negotiator 2 obtains $\delta^t f(\max\{f^{-1}(y^{**}), \delta x\})$. Since $\delta x \leq \delta x^R$, $\delta f(\delta x) \leq f(x)$ holds by the inequality (1). Then, we obtain $\delta f(\max\{f^{-1}(y^{**}), \delta x\}) \leq \delta f(\delta x) \leq f(x)$, that is, $\delta^t f(\max\{f^{-1}(y^{**}), \delta x\}) \leq \delta^{t-1} f(x)$. Therefore, negotiator 2 cannot improve her payoff by demanding y^{**} ($> f(x)$).

Consequently, we can find that the SSPE σ is consistent. Therefore, the outcome $(p(x), 1)$ such that $x \in (f^{-1}(\delta f(\alpha)), x^R]$ is supported as an SSPE outcome.

5. Suppose that, in some SSPE σ, negotiators 1 and 2 demand x and $f(x)$ such that $x \in (x^R, \overline{x}]$, respectively. Consider the case that negotiator 2 deviates from σ and demands $f(\delta x)$. Then, the game proceeds to the next period and the mediator proposes some $p(z)$ such that $z \in [\delta x, x]$ or chooses pass. Now, $A(x, x, f(\delta x))$ can be transformed as $A(x, x, f(\delta x)) = [\delta x, x] \cap [\delta x, f^{-1}(\delta f(x))] = [\delta x, x]$ ($\neq \emptyset$). Since $\alpha < \delta x^R < \delta x$, the mediator proposes $p(\delta x)$ by the case 2 of Lemma 2. Therefore, negotiator 2 obtains $\delta^t f(\delta x)$.

   Since $\delta x > \delta x^R$, we obtain $\delta f(\delta x) > f(x)$ by the inequality (2). Then, $\delta^t f(\delta x) > \delta^{t-1} f(x)$. Therefore, negotiator 2 can improve her payoff by deviating from σ. This is a contradiction. Consequently, the outcome $(p(x), 1)$ such that $x \in (x^R, \overline{x}]$ is not supported as an SSPE outcome.

□

Roughly, Lemma 3 can be explained as follows. Consider the case where negotiators 1 and 2 demand x and $f(x)$, respectively, in some stationary strategy σ. When the negotiators deviate from σ, we can easily confirm that each negotiator cannot improve her payoff by proposing some demand smaller than the demand under σ. Therefore, it is sufficient to consider the case where the negotiators propose larger demands. Then, the game proceeds to the next period.

First, consider the case $x < \alpha$. In this case, even if negotiator 2 deviates from σ and proposes larger demand, the mediator proposes $p(x)$ at the next period. Therefore, negotiator 2 cannot improve her payoff by deviating from σ. Conversely, consider the case where negotiator 1 deviates from σ and proposes sufficiently large demand x^*. Then, the game proceeds to the next period and $\max A(x, x^*, f(x)) = f^{-1}(\delta f(x))$. Here, notice that the mediator's proposal which gives negotiator 1 a utility larger than $f^{-1}(\delta f(x))$ is rejected by negotiator 2. Therefore, the mediator proposes $p(\min\{f^{-1}(\delta f(x)), \alpha\})$ (notice that $\min A(x, x^*, f(x)) = x < \alpha$). When $x \in [0, \delta\alpha)$, reaching an agreement with the mediator's proposal $p(\min\{f^{-1}(\delta f(x)), \alpha\})$ at the next period is more profitable for negotiator 1 than reaching an agreement with $p(x)$ at the current period. Therefore, negotiator 1 deviates from σ, that is, σ is not an SSPE. When $x \in [\delta\alpha, \alpha)$, it is converse. Therefore, negotiator 1 does not deviate from σ, that is, σ is an SSPE.

The case $x \geq \alpha$ is similarly explained. In this case, negotiator 1 cannot improve her payoff by deviating from σ and proposing larger demands. When negotiator 2 deviates from σ and proposes sufficiently large demand y^*, the game proceeds to the next period and $\min A(x, x, y^*) = \delta x$. Then, the mediator proposes $p(\max\{\delta x, \alpha\})$ at the next period (notice that $\max A(x, x, y^*) =$

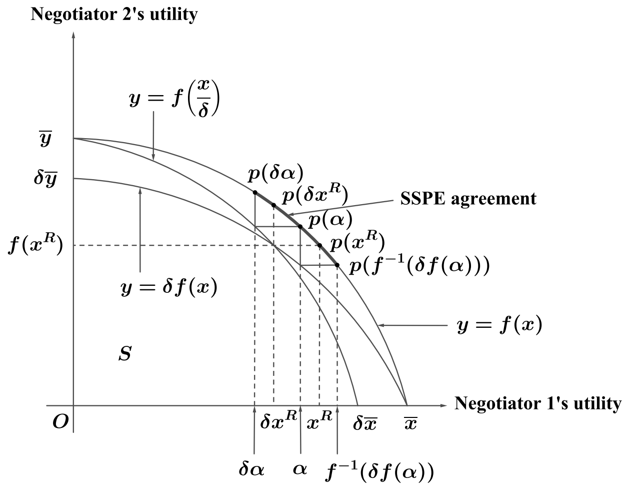

Figure 7: SSPE agreement at period 1 when $\alpha \in [\delta x^R, x^R]$

$x \geq \alpha$). When $x \in [\alpha, x^R]$, negotiator 2 does not deviate from σ, that is, σ is an SSPE. When $x \in (x^R, \overline{x}]$, negotiator 2 deviates from σ, that is, σ is not an SSPE.

In the following, we analyze the cases of $\alpha \in [\delta x^R, x^R]$ and $\alpha \in (x^R, \overline{x}]$. Although the regions of SSPE agreements in these cases are different from the case of $\alpha \in [0, \delta x^R)$, the above discussion is similarly applied to these cases. Now, we analyze the case of $\alpha \in [\delta x^R, x^R]$.

Lemma 4. *When $\alpha \in [\delta x^R, x^R]$, the outcome $(p(x), 1)$ is supported as an SSPE outcome if and only if $x \in [\delta\alpha, f^{-1}(\delta f(\alpha))]$ (see Figure 7). In the SSPE, negotiators 1 and 2 demand x and $f(x)$, respectively, and the mediator follows the strategy described in Lemma 2.*

Proof. Without loss of generality, in the following proofs, we consider that the negotiators propose their demands at period t and the mediator proposes at period $t+1$.

1. Suppose that, in some SSPE σ, negotiators 1 and 2 demand x and $f(x)$ such that $x \in [0, \delta\alpha)$, respectively. Consider the case that negotiator 1 deviates from σ and demands $f^{-1}(\delta f(x))$. By the same proof in the case 1 of Lemma 3, we can find that the outcome $(p(x), 1)$ such that $x \in [0, \delta\alpha)$ is not supported as an SSPE outcome.

2. Suppose that, in some SSPE σ, negotiators 1 and 2 demand x and $f(x)$ such that $x \in [\delta\alpha, \alpha)$, respectively. Then, the mediator's proposals in σ are described in Lemma 2. By the same proof in the case 2 of Lemma 3, we can find that the outcome $(p(x), 1)$ such that $x \in [\delta\alpha, \alpha)$ is supported as an SSPE outcome.

3. Suppose that, in some SSPE σ, negotiators 1 and 2 demand x and $f(x)$ such that $x \in [\alpha, f^{-1}(\delta f(\alpha))]$, respectively. Then, the mediator's proposals in σ are described in Lemma 2. By the same proof in the case 3 of Lemma 3, we can find that the outcome $(p(x), 1)$ such that $x \in [\alpha, f^{-1}(\delta f(\alpha))]$ is supported as an SSPE outcome.

4. Suppose that, in some SSPE σ, negotiators 1 and 2 demand x and $f(x)$ such that $x \in (f^{-1}(\delta f(\alpha)), \overline{x}]$, respectively. Consider the case that negotiator 2 deviates from σ and demands $f(\delta x)$. Then, the game proceeds to the next period and the mediator proposes

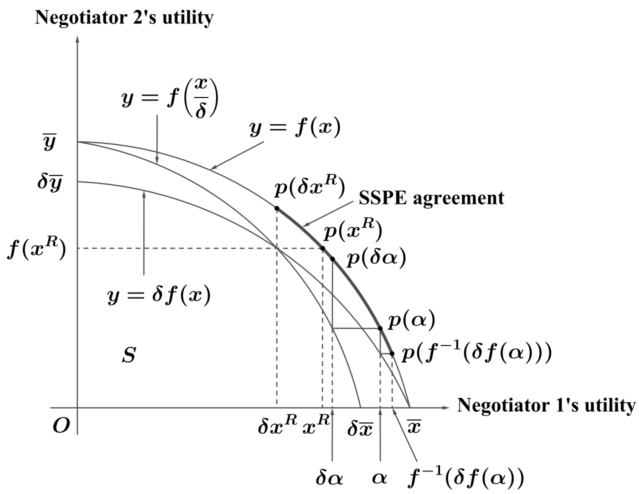

Figure 8: SSPE agreement at period 1 when $\alpha \in (\delta x^R, \overline{x}]$

some $p(z)$ such that $z \in [\delta x, x]$ or chooses pass. Now, $A(x, x, \delta x)$ can be transformed as $A(x, x, \delta x) = [\delta x, x] \cap [\delta x, f^{-1}(\delta f(x))] = [\delta x, x]$ $(\neq \emptyset)$.

If $\alpha \geq \delta x$, since $\alpha < f^{-1}(\delta f(\alpha)) < x$, the mediator proposes $p(\alpha)$ by the case 1 of Lemma 2, and negotiator 2 obtains $\delta^t f(\alpha)$. Since $f^{-1}(\delta f(\alpha)) < x$, we obtain $\delta^t f(\alpha) > \delta^{t-1} f(x)$. If $\alpha < \delta x$, the mediator proposes $p(\delta x)$ by the case 2 of Lemma 2, and negotiator 2 obtains $\delta^t f(\delta x)$. Since $\delta x > \alpha \geq \delta x^R$, we obtain $\delta f(\delta x) > f(x)$ by the inequality (2). Thus, $\delta^t f(\delta x) > \delta^{t-1} f(x)$.

Therefore, negotiator 2 can improve her payoff by deviating from σ and demanding $f(\delta x)$. This is a contradiction. Thus, the outcome $(p(x), 1)$ such that $x \in (f^{-1}(\delta f(\alpha)), \overline{x}]$ is not supported as an SSPE outcome.

□

Finally, we analyze the case of $\alpha \in (x^R, \overline{x}]$.

Lemma 5. *When $\alpha \in (x^R, \overline{x}]$, the outcome $(p(x), 1)$ is supported as an SSPE outcome if and only if $x \in [\delta x^R, f^{-1}(\delta f(\alpha))]$ (see Figure 8). In the SSPE, negotiators 1 and 2 demand x and $f(x)$, respectively, and the mediator follows the strategy described in Lemma 2.*

Proof. By exchanging the roles of negotiators 1 and 2 in Lemma 3, we can prove Lemma 5. □

By summarizing Lemma 3, 4, and 5, SSPE agreements at period 1 are described as follows.

Theorem 1. *The outcome $(p(x), 1)$ is supported as an SSPE outcome if and only if*

1. *$x \in [\delta\alpha, x^R(\delta)]$ when $\alpha \in [0, \delta x^R(\delta))$,*
2. *$x \in [\delta\alpha, f^{-1}(\delta f(\alpha))]$ when $\alpha \in [\delta x^R(\delta), x^R(\delta)]$, and*
3. *$x \in [\delta x^R(\delta), f^{-1}(\delta f(\alpha))]$ when $\alpha \in (x^R(\delta), \overline{x}]$.*

In the SSPE, negotiators 1 and 2 demand x and $f(x)$, respectively, and the mediator follows the strategy described in Lemma 2.

Since $\delta\alpha \leq \delta x^R(\delta)$ holds when $\alpha \in [0, x^R(\delta)]$ and $x^R(\delta) \leq f^{-1}(\delta f(\alpha))$ holds when $\alpha \in [\delta x^R(\delta), \overline{x}]$, an agreement on $\partial S^R = \{p(x) \mid x \in [\delta x^R(\delta), x^R(\delta)]\}$ is supported as an SSPE agreement for any $\alpha \in [0, \overline{x}]$. Now, notice that $p(\delta x^R(\delta))$ and $p(x^R(\delta))$ are the negotiators' SPE offers in the Rubinstein's alternating-offers model. Since it is well known that the NBS of the bargaining problem (S, d) lies on ∂S^R (for example, see Osborne and Rubinstein (1994)), we can find that, for all $\alpha \in [0, \overline{x}]$, the outcome that the negotiators reach an agreement with the NBS at period 1 is supported as an SSPE outcome.

3.2 SSPE agreement at even periods

Next, we analyze SSPE outcomes such that the mediator's proposal is accepted at even periods. By the definition of SSPE, in such SSPE outcomes, the mediator's proposal is accepted at period 2. In this SSPE, the agreement is delayed. First, we describe the mediator's strategy in such an SSPE.

Lemma 6. *Suppose that the mediator's proposal $p(x)$ is accepted at even periods in some SSPE σ. Then, in the path after negotiators 1 and 2 demand x' and y', respectively (where $(x', y') \notin S$), the mediator chooses the following action at even periods under σ. Now, we define $B(x, x', y') = [f^{-1}(y'), x'] \cap [\delta^2 x, f^{-1}(\delta^2 f(x))]$.*

1. *When $\alpha \in B(x, x', y')$, the mediator proposes $p(\alpha) \in \partial S$ (it is accepted by both negotiators).*
2. *When $B(x, x', y') \neq \emptyset$, $\alpha < \min B(x, x', y')$, and $\min B(x, x', y') \leq x$, the mediator proposes $p(\min B(x, x', y')) \in \partial S$ (it is accepted by both negotiators).*
3. *When $B(x, x', y') \neq \emptyset$, $\alpha < \min B(x, x', y')$, and $\min B(x, x', y') > x$, the mediator proposes $p(\min B(x, x', y')) \in \partial S$ if $u(p(\min B(x, x', y'))) > \delta^2 u(p(x))$ (it is accepted by both negotiators), and chooses pass (or offers some proposal rejected by some negotiator) if $u(p(\min B(x, x', y'))) < \delta^2 u(p(x))$. If $u(p(\min B(x, x', y'))) = \delta^2 u(p(x))$, the mediator proposes $p(\min B(x, x', y'))$ or chooses pass (or offers some proposal rejected by some negotiator).*
4. *When $B(x, x', y') \neq \emptyset$, $\alpha > \max B(x, x', y')$, and $\max B(x, x', y') \geq x$, the mediator proposes $p(\max B(x, x', y')) \in \partial S$ (it is accepted by both negotiators).*
5. *When $B(x, x', y') \neq \emptyset$, $\alpha > \max B(x, x', y')$, and $\max B(x, x', y') < x$, the mediator proposes $p(\max B(x, x', y')) \in \partial S$ if $u(p(\max B(x, x', y'))) > \delta^2 u(p(x))$ (it is accepted by both negotiators), and chooses pass (or offers some proposal rejected by some negotiator) if $u(p(\max B(x, x', y'))) < \delta^2 u(p(x))$. If $u(p(\max B(x, x', y'))) = \delta^2 u(p(x))$, the mediator proposes $p(\max B(x, x', y'))$ or chooses pass (or offers some proposal rejected by some negotiator).*
6. *When $B(x, x', y') = \emptyset$, the mediator proposes some $p(z) \in \partial S$ satisfying $z \in [f^{-1}(y'), x']$ (it is rejected by some negotiator) or chooses pass.*

Proof. The proof is analogous to Lemma 2. □

By using Lemma 6, we derive SSPE agreement at period 2.

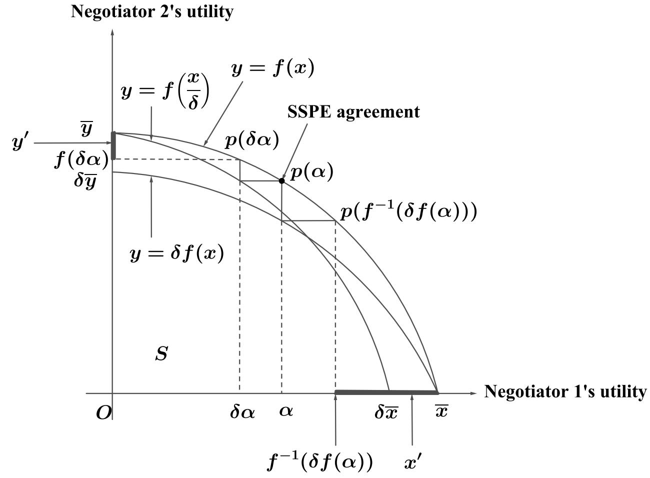

Figure 9: SSPE agreement at period 2

Theorem 2. *The outcome* $(p(x), 2)$ *is supported as an SSPE outcome if and only if* $x = \alpha$. *In the SSPE, negotiators 1 and 2 propose some* $x' \in [f^{-1}(\delta f(\alpha)), \overline{x}]$ *and* $y' \in [f(\delta\alpha), \overline{y}]$*, respectively, and the mediator follows the strategy described in Lemma 6. (See Figure 9.)*

Proof. Suppose that σ is the SSPE such that negotiators 1 and 2 demand x' and y' ($(x', y') \notin S$), respectively, and the mediator's proposal $p(x)$ is accepted after x' and y' are demanded. Without loss of generality, in the following proofs, we consider that the negotiators propose their demands at period t and the mediator proposes at period $t+1$.

1. Suppose $x = \alpha$. Since the mediator's proposal $p(\alpha)$ is accepted under σ, $\alpha \in B(\alpha, x', y')$ must hold by Lemma 6. Then, $f^{-1}(y') \leq \alpha \leq x'$, that is, $x' \geq \alpha$ and $y' \geq f(\alpha)$ must hold.

   Now, suppose $x' \in [\alpha, f^{-1}(\delta f(\alpha)))$. Consider the case that negotiator 2 deviates from σ and demands $f(x')$. Then, negotiator 2 obtains $\delta^{t-1} f(x')$ ($> \delta^t f(\alpha)$). Therefore, negotiator 2 can improve her payoff by deviating from σ and demanding $f(x')$. Also, suppose $y' \in [f(\alpha), f(\delta\alpha))$. Consider the case that negotiator 1 deviates from σ and demands $f^{-1}(y')$. Then, negotiator 1 obtains $\delta^{t-1} f^{-1}(y')$ ($> \delta^t \alpha$). Therefore, negotiator 1 can improve her payoff by deviating from σ and demanding $f^{-1}(y')$. Thus, x' and y' must satisfy $x' \in [f^{-1}(\delta f(\alpha)), \overline{x}]$ and $y' \in [f(\delta\alpha), \overline{y}]$, respectively.

   (a) We prove that negotiator 1 cannot improve her payoff by deviating from σ when $x' \in [f^{-1}(\delta f(\alpha)), \overline{x}]$ and $y' \in (f(\delta^2\alpha), \overline{y}]$.

   Consider the case that negotiator 1 deviates from σ and demands $x^* \in [0, f^{-1}(y')]$. Then, since $(x^*, y') \in S$, negotiator 1 obtains $\delta^{t-1} x^*$. Also, since $x^* \leq f^{-1}(y') < \delta^2\alpha < \delta\alpha$, we obtain $\delta^{t-1} x^* < \delta^t \alpha$. Therefore, negotiator 1 cannot improve her payoff by deviating from σ and demanding x^*.

   Consider the case that negotiator 1 deviates from σ and demands $x^{**} \in (f^{-1}(y'), \delta^2\alpha)$. Then, the game proceeds to the next period and the mediator proposes some $p(z)$ such that $z \in [f^{-1}(y'), x^{**}]$ or chooses pass. Now, since $x^{**} \in (f^{-1}(y'), \delta^2\alpha)$, $B(\alpha, x^{**}, y') =$

$[f^{-1}(y'), x^{**}] \cap [\delta^2\alpha, f^{-1}(\delta^2 f(\alpha))] = \emptyset$. Therefore, the mediator's proposal is not accepted and the game proceeds to the next period. Then, negotiator 1 obtains $\delta^{t+2}\alpha$ at period $t+3$ under σ. Since $\delta^{t+2}\alpha < \delta^t\alpha$, negotiator 1 cannot improve her payoff by deviating from σ and demanding x^{**}.

Consider the case that negotiator 1 deviates from σ and demands $x^{***} \in [\delta^2\alpha, \overline{x}]$. Then, the game proceeds to the next period and the mediator proposes some $p(z)$ such that $z \in [f^{-1}(y'), x^{***}]$ or chooses pass. Now, since $x^{***} \in [\delta^2\alpha, \overline{x}]$ and $y' \in (f(\delta^2\alpha), \overline{y}]$, $B(\alpha, x^{***}, y') = [f^{-1}(y'), x^{***}] \cap [\delta^2\alpha, f^{-1}(\delta^2 f(\alpha))] = [\delta^2\alpha, \min\{x^{***}, f^{-1}(\delta^2 f(\alpha))\}]$ $(\neq \emptyset)$. If $\alpha \leq \min\{x^{***}, f^{-1}(\delta^2 f(\alpha))\}$, the mediator proposes $p(\alpha)$ by the case 1 of Lemma 6, and negotiator 1 obtains $\delta^t\alpha$. If $\alpha > \min\{x^{***}, f^{-1}(\delta^2 f(\alpha))\}$, the mediator proposes $p(\min\{x^{***}, f^{-1}(\delta^2 f(\alpha))\})$ or chooses pass by the case 5 of Lemma 6. Then, negotiator 1 obtains $\delta^t \min\{x^{***}, f^{-1}(\delta^2 f(\alpha))\}$ at period $t+1$ or $\delta^{t+2}\alpha$ at period $t+3$. Now, $\delta^t \min\{x^{***}, f^{-1}(\delta^2 f(\alpha))\} < \delta^t\alpha$ and $\delta^{t+2}\alpha < \delta^t\alpha$ hold. Therefore, negotiator 1 cannot improve her payoff by deviating from σ and demanding x^{***}.

By the above discussion, we can find that negotiator 1 cannot improve her payoff by deviating from σ when $x' \in [f^{-1}(\delta f(\alpha)), \overline{x}]$ and $y' \in (f(\delta^2\alpha), \overline{y}]$.

(b) We prove that negotiator 1 cannot improve her payoff by deviating from σ when $x' \in [f^{-1}(\delta f(\alpha)), \overline{x}]$ and $y' \in [f(\delta\alpha), f(\delta^2\alpha)]$.

Consider the case that negotiator 1 deviates from σ and demands $x^* \in [0, f^{-1}(y')]$. Then, since $(x^*, y') \in S$, negotiator 1 obtains $\delta^{t-1}x^*$. Also, since $x^* \leq f^{-1}(y') \leq \delta\alpha$, we obtain $\delta^{t-1}x^* \leq \delta^t\alpha$. Therefore, negotiator 1 cannot improve her payoff by deviating from σ and demanding x^*.

Consider the case that negotiator 1 deviates from σ and demands $x^{**} \in (f^{-1}(y'), \overline{x}]$. Then, the game proceeds to the next period and the mediator proposes some $p(z)$ such that $z \in [f^{-1}(y'), x^{**}]$ or chooses pass. Now, since $y' \in [f(\delta\alpha), f(\delta^2\alpha)]$, we obtain $f(\delta^2\alpha) \geq y' \geq f(\delta\alpha) > \delta^2 f(\alpha)$. Therefore, $B(\alpha, x^{**}, y') = [f^{-1}(y'), x^{**}] \cap [\delta^2\alpha, f^{-1}(\delta^2 f(\alpha))] = [f^{-1}(y'), \min\{x^{**}, f^{-1}(\delta^2 f(\alpha))\}]$ $(\neq \emptyset)$. If $\alpha \leq \min\{x^{**}, f^{-1}(\delta^2 f(\alpha))\}$, since $f^{-1}(y') \leq \delta\alpha < \alpha$, the mediator proposes $p(\alpha)$ by the case 1 of Lemma 6, and negotiator 1 obtains $\delta^t\alpha$. If $\alpha > \min\{x^{**}, f^{-1}(\delta^2 f(\alpha))\}$, the mediator proposes $p(\min\{x^{**}, f^{-1}(\delta^2 f(\alpha))\})$ or chooses pass by the case 5 of Lemma 6. Then, negotiator 1 obtains $\delta^t \min\{x^{**}, f^{-1}(\delta^2 f(\alpha))\}$ at period $t+1$ or $\delta^{t+2}\alpha$ at period $t+3$. Now, $\delta^t \min\{x^{**}, f^{-1}(\delta^2 f(\alpha))\} < \delta^t\alpha$ and $\delta^{t+2}\alpha < \delta^t\alpha$ hold. Therefore, negotiator 1 cannot improve her payoff by deviating from σ and demanding x^{**}.

By the above discussion, we can find that negotiator 1 cannot improve her payoff by deviating from σ when $x' \in [f^{-1}(\delta f(\alpha)), \overline{x}]$ and $y' \in [f(\delta\alpha), f(\delta^2\alpha)]$.

Summarizing (a) and (b), we can find that negotiator 1 cannot improve her payoff by deviating from σ when $x' \in [f^{-1}(\delta f(\alpha)), \overline{x}]$ and $y' \in [f(\delta\alpha), \overline{y}]$. By exchanging the roles of negotiators 1 and 2 in the above proofs, we can also prove that negotiator 2 cannot improve her payoff by deviating from σ when $x' \in [f^{-1}(\delta f(\alpha)), \overline{x}]$ and $y' \in [f(\delta\alpha), \overline{y}]$. Therefore, the strategy profile where negotiators 1 and 2 propose x' and y' such that $x' \in [f^{-1}(\delta f(\alpha)), \overline{x}]$ and $y' \in [f(\delta\alpha), \overline{y}]$, respectively, and the mediator follows the strategy described in Lemma 6 is an SSPE. Under this SSPE, after x' and y' are demanded by the negotiators, the mediator proposes $p(\alpha)$ and it is accepted.

2. Suppose $x \in [0, \alpha)$. That is, in the SSPE σ, the mediator's proposal $p(x)$ such that $x < \alpha$ is accepted. Then, $x \in B(x, x', y')$ must hold. Now, we prove $x = \max B(x, x', y')$. Suppose $x < \max B(x, x', y')$. Then, if $\alpha > \max B(x, x', y')$, the mediator proposes $p(\max B(x, x', y'))$ under σ by the case 4 of Lemma 6. If $\alpha \leq \max B(x, x', y')$, since

$\alpha > x \geq \min B(x, x', y')$, the mediator proposes $p(\alpha)$ under σ by the case 1 of Lemma 6. This contradicts to the fact that the mediator proposes $p(x)$ such that $x < \max B(x, x', y')$ and $x < \alpha$ under σ. Therefore, x must satisfy $x = \max B(x, x', y')$ $(< \alpha)$. Then, since $B(x, x', y') = [f^{-1}(y'), x'] \cap [\delta^2 x, f^{-1}(\delta^2 f(x))]$ and $\delta^2 x < x$ $(= \max B(x, x', y')) < f^{-1}(\delta^2 f(x))$, x' must satisfy $x' = \max B(x, x', y') = x$.

Consider the case that negotiator 1 deviates from σ and demands $f^{-1}(\delta^2 f(x))$ $(> x = x')$. Then, the game proceeds to the next period and the mediator proposes some $p(z)$ such that $z \in [f^{-1}(y'), f^{-1}(\delta^2 f(x))]$ or chooses pass. Now, $B(x, f^{-1}(\delta^2 f(x)), y')$ can be transformed as $B(x, f^{-1}(\delta^2 f(x)), y') = [f^{-1}(y'), f^{-1}(\delta^2 f(x))] \cap [\delta^2 x, f^{-1}(\delta^2 f(x))] = [\max\{f^{-1}(y'), \delta^2 x\}, f^{-1}(\delta^2 f(x))]$ $(\neq \emptyset)$.

If $\alpha \leq f^{-1}(\delta^2 f(x))$, since $f^{-1}(y') < x' = x < \alpha$ and $\delta^2 x < x < \alpha$ hold, the mediator proposes $p(\alpha)$ by the case 1 of Lemma 6, and negotiator 1 obtains $\delta^t \alpha$ $(> \delta^t x)$. If $\alpha > f^{-1}(\delta^2 f(x))$, the mediator proposes $p(f^{-1}(\delta^2 f(x)))$ by the case 4 of Lemma 6, and negotiator 1 obtains $\delta^t f^{-1}(\delta^2 f(x))$ $(> \delta^t x)$. Therefore, negotiator 1 can improve her payoff by deviating from σ and demanding $f^{-1}(\delta^2 f(x))$. This is a contradiction. Thus, the outcome $(p(x), 2)$ such that $x \in [0, \alpha)$ is not supported as an SSPE outcome.

3. Suppose $x \in (\alpha, \overline{x}]$. By the proof analogous to the case 2, we can find that the outcome $(p(x), 2)$ such that $x \in (\alpha, \overline{x}]$ is not supported as an SSPE outcome.

□

The SSPE of Theorem 2 can be interpreted as follows. In this SSPE, since the negotiators' demands are incompatible, they cannot reach an agreement by themselves. Thus, the mediator proposes a plan of an agreement to facilitate the reaching of an agreement. If some negotiator rejects the mediator's proposal, the negotiation breaks down and they cannot make a profit. Then, since accepting the mediator's proposal is better than disagreement for both negotiators, they decide to accept the mediator's proposal.

3.3 Disagreement

In this subsection, we analyze disagreement and obtain the following theorem.

Theorem 3. *Disagreement is not supported as an SSPE outcome.*

Proof. Suppose that disagreement occurs in some SSPE σ. Then, negotiators 1, 2, and the mediator obtain payoffs of zero. Since disagreement occurs, in the SSPE σ, negotiators 1 and 2 propose x' and y' such that $(x', y') \notin S$, respectively. After x' and y' are demanded, the mediator proposes some $p(z)$ such that $z \in [f^{-1}(y'), x']$ or chooses pass. Then, there exists some $z' \in [f^{-1}(y'), x']$ such that $z' > 0$ and $f(z') > 0$. Therefore, the proposal $p(z')$ is accepted by the negotiators. Now, $u(p(z')) > 0$ by $u(0, f(0)) \geq 0$, $u(\overline{x}, f(\overline{x})) \geq 0$, and Assumption 1. Thus, the mediator can obtain a payoff larger than zero by deviating from σ and proposing $p(z')$. This is a contradiction. Hence, disagreement is not supported as an SSPE outcome. □

Theorem 3 implies that the mediator can resolve conflict. As the same as Theorem 2, this result is caused by the fact that accepting the mediator's proposal is better than disagreement for both negotiators.

3.4 Agreement with the NBS

In this subsection, we analyze properties of an agreement with the NBS. Let $p(x^N) = (x^N, f(x^N))$ be the NBS of the bargaining problem (S, d). In subsection 3.1, we saw that an agreement with the NBS at period 1 is always supported as an SSPE outcome. Now, we can derive a stronger result that an agreement with the NBS at period 1 is the "unique" outcome which is supported as an SSPE outcome for all $\delta \in (0, 1)$ and for all $\alpha \in [0, \overline{x}]$. To derive this result, first, we prove the following proposition by Theorem 1.

Proposition 1. *An agreement with the NBS is the unique agreement which is supported as an SSPE agreement at period 1 for all $\delta \in (0, 1)$ and for all $\alpha \in [0, \overline{x}]$.*

Proof. First, notice that, since $p(\delta x^R(\delta))$ and $p(x^R(\delta))$ are the negotiators' SPE offers in the Rubinstein's alternating-offers model, $\lim_{\delta \uparrow 1} \delta x^R(\delta) = x^N$ and $\lim_{\delta \uparrow 1} x^R(\delta) = x^N$ hold (for example, see Binmore et al. (1986) and Osborne and Rubinstein (1994)).

Suppose $\alpha < x^N$. Since $\lim_{\delta \uparrow 1} \delta x^R(\delta) = x^N$, there exists some δ' such that $\alpha < \delta x^R(\delta)$ holds for all $\delta \in (\delta', 1)$. Then, by the case 1 of Theorem 1, for $\delta \in (\delta', 1)$, an agreement with $p(x)$ is an SSPE agreement if and only if $x \in [\delta\alpha, x^R(\delta)]$. Now, since $\lim_{\delta \uparrow 1} x^R(\delta) = x^N$, for $x > x^N$, there exists some $\delta^* \in (\delta', 1)$ such that $x > x^R(\delta^*)$. Therefore, an agreement with $p(x)$ such that $x > x^N$ is not supported as an SSPE agreement for some $\alpha \in [0, x^N)$ and $\delta^* \in (\delta', 1)$. This implies that, if $p(x)$ is supported as an SSPE agreement at period 1 for all $\delta \in (0, 1)$ and $\alpha \in [0, \overline{x}]$, x must satisfy $x \le x^N$. Conversely, suppose $\alpha > x^N$. By the proof analogous to the case of $\alpha < x^N$, we can prove that, if $p(x)$ is supported as an SSPE agreement at period 1 for all $\delta \in (0, 1)$ and $\alpha \in [0, \overline{x}]$, x must satisfy $x \ge x^N$ (by the case 3 of Theorem 1). Therefore, if $p(x)$ is supported as an SSPE agreement at period 1 for all $\delta \in (0, 1)$ and $\alpha \in [0, \overline{x}]$, x must satisfy $x = x^N$. Then, since $p(x^N)$ is an SSPE agreement at period 1 for all $\delta \in (0, 1)$ and $\alpha \in [0, \overline{x}]$, we obtain Proposition 1. $\square$

Now, by Theorem 2, we can see that there is no agreement which is supported as an SSPE agreement at period 2 for all $\alpha \in [0, \overline{x}]$. Therefore, by combining Theorem 2, 3, and Proposition 1, we immediately obtain the following result.

Theorem 4. *The outcome $(p(x^N), 1)$ is the unique outcome which is supported as an SSPE outcome for all $\delta \in (0, 1)$ and for all $\alpha \in [0, \overline{x}]$.*

Even if the mediator is biased, the NBS can always be achieved in SSPE. Now, the question is: Is there any other agreement which can always be achieved in SSPE? The result of Theorem 4 denies the existence of such an agreement. That is, an agreement other than the NBS may be eliminated from SSPE agreement. In contrast, the fair agreement in the sense of the NBS is the unique agreement which is always supported as an SSPE agreement.

Next, we consider the case where δ approaches to one. First, as a corollary of Theorem 1, 2, and 3, we obtain the following result. Notice that $\lim_{\delta \uparrow 1} \delta\alpha = \lim_{\delta \uparrow 1} f^{-1}(\delta f(\alpha)) = \alpha$ and $\lim_{\delta \uparrow 1} \delta x^R(\delta) = \lim_{\delta \uparrow 1} x^R(\delta) = x^N$.

Corollary 1. *The outcome $(p(x), 1)$ is supported as an SSPE outcome under $\delta \uparrow 1$ if and only if*

1. *$x \in [\alpha, x^N]$ when $\alpha \in [0, x^N)$,*
2. *$x = x^N$ when $\alpha = x^N$, and*
3. *$x \in [x^N, \alpha]$ when $\alpha \in (x^N, \overline{x}]$.*

The outcome $(p(x), 2)$ is supported as an SSPE outcome under $\delta \uparrow 1$ if and only if $x = \alpha$. Also, disagreement is not supported as an SSPE outcome.

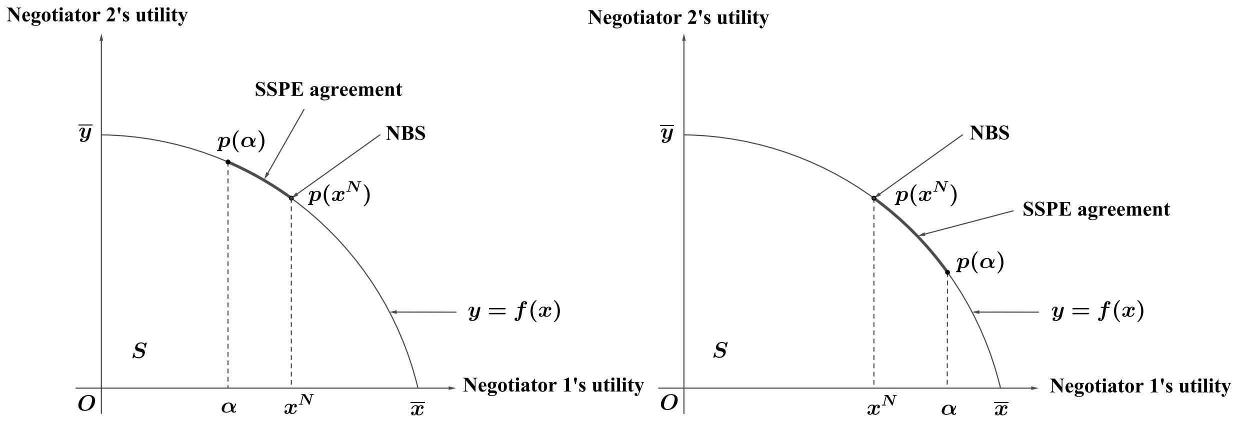

Figure 10: SSPE agreement when $\delta \uparrow 1$

This result shows that, when δ approaches to one, an agreement with $p(x)$ is an SSPE agreement if and only if $p(x)$ lies between the NBS $p(x^N)$ and the mediator's ideal agreement $p(\alpha)$ (see Figure 10). That is, as the mediator's ideal agreement approaches to the NBS, the set of SSPE agreements shrinks. Therefore, when the mediator is sufficiently fair, the agreement achieved in SSPE is sufficiently close to the NBS. Especially, when the mediator wishes to achieve the NBS, we obtain the following desirable result.

Theorem 5. *When $p(\alpha) = p(x^N)$, SSPE outcomes under $\delta \uparrow 1$ are $(p(x^N), 1)$ and $(p(x^N), 2)$. Therefore, when $\delta \uparrow 1$, the NBS is the unique agreement achieved in SSPEs.*

This result shows that, when the mediator wishes to achieve the NBS, the fair agreement (the NBS) is "surely" achieved in SSPEs under $\delta \uparrow 1$. That is, the fair mediator facilitates the reaching of a fair agreement.

4 Comparison with simultaneous-offers bargaining without a mediator and with an arbitrator

In this section, we compare the model with a mediator with models without a mediator and with an arbitrator. By comparison, we analyze how the mediator affects the bargaining outcome. In the following, we use the same notation as the model with a mediator.

First, we compare the model with a mediator with a model without a mediator. The model without a mediator is as follows. The game starts from period 1. At period t, negotiators 1 and 2 simultaneously propose their demands $x \in [0, \overline{x}]$ and $y \in [0, \overline{y}]$, respectively. If $(x, y) \in S$, then the game ends and negotiators 1 and 2 receive $\delta^{t-1}x$ and $\delta^{t-1}y$, respectively. If $(x, y) \notin S$, the game proceeds to the next period $t + 1$ and repeat the above process. The game continues until an agreement is reached. We derive SSPE outcomes of this model (that is, derive outcomes induced by the SPE such that each negotiator's demand is always the same value) and obtain the following result.

Proposition 2. *In the model without a mediator, for all $x \in [0, \overline{x}]$, an agreement with $p(x)$ is supported as an SSPE outcome (see Figure 11) and disagreement is supported as an SSPE outcome.*

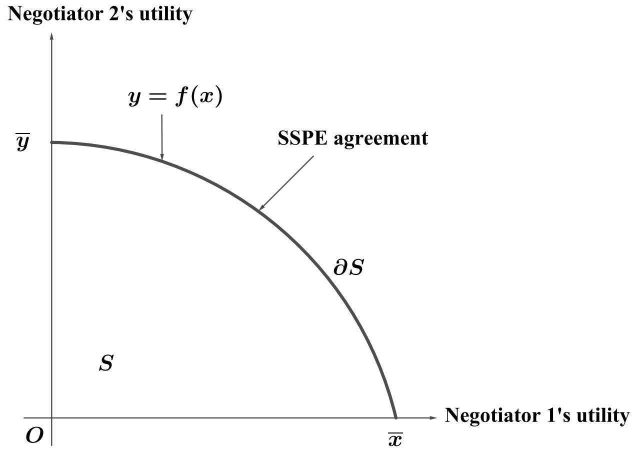

Figure 11: SSPE agreement in the model without a mediator

Proof. Without loss of generality, in the following proof, we consider that the negotiators propose their demands at period t.

Consider the stationary strategy profile $\sigma(x)$ where negotiators 1 and 2 demand x and $f(x)$, respectively. Then, negotiators 1 and 2 receive payoffs $\delta^{t-1}x$ and $\delta^{t-1}f(x)$, respectively. If negotiator 1 deviates from $\sigma(x)$ and demands x^* such that $x^* < x$, she obtains $\delta^{t-1}x^*$ $(< \delta^{t-1}x)$. If negotiator 1 deviates from $\sigma(x)$ and demands x^{**} such that $x^{**} > x$, she obtains $\delta^t x$ $(< \delta^{t-1}x)$ at the next period. Therefore, negotiator 1 cannot improve her payoff by deviating from $\sigma(x)$. Also, negotiator 2 cannot improve her payoff by deviating from $\sigma(x)$. Consequently, for all $x \in [0, \overline{x}]$, an agreement with $p(x)$ is supported as an SSPE outcome.

Next, consider the stationary strategy profile σ^d where negotiators 1 and 2 demand $\overline{x}$ and $f(0)$, respectively. Then, disagreement occurs and each negotiator receives a payoff of zero. Even if negotiator 1 deviates from σ^d and demands $x^* \in [0, \overline{x})$, she obtains a payoff of zero. Therefore, negotiator 1 cannot improve her payoff by deviating from σ^d. Also, negotiator 2 cannot improve her payoff by deviating from σ^d. Consequently, disagreement is supported as an SSPE outcome. $\square$

Disagreement is supported as an SSPE outcome in the model without a mediator, but it does not appear as an SSPE outcome in the model with a mediator. These results imply that the mediator has the power to resolve conflict. Also, in the model without a mediator, since all agreements on the Pareto frontier ∂S can be achieved as an SSPE agreement, an unfair agreement may be achieved. In contrast to it, in the model with a mediator, when the mediator is sufficiently fair, the agreement achieved in SSPE is sufficiently close to the fair agreement (the NBS).

Next, we compare the model with a mediator with a model with an arbitrator. The role of an arbitrator is imposing some agreement as a final bargaining outcome when negotiators cannot reach an agreement by themselves. The model with an arbitrator is as follows. At period 1, negotiators 1 and 2 simultaneously propose their demands $x \in [0, \overline{x}]$ and $y \in [0, \overline{y}]$, respectively. If $(x, y) \in S$, then the game ends and negotiators 1 and 2 receive x and y, respectively. If $(x, y) \notin S$, the game proceeds to period 2. At period 2, the arbitrator imposes some $p(z)$ such that $z \in [f^{-1}(y), x]$ as an outcome of the bargaining. When the arbitrator imposes $p(z)$, negotiators 1, 2, and the arbitrator receive δz, $\delta f(z)$, and $\delta u(p(z))$, respectively, where $u : S \to \mathbb{R}_+$ is

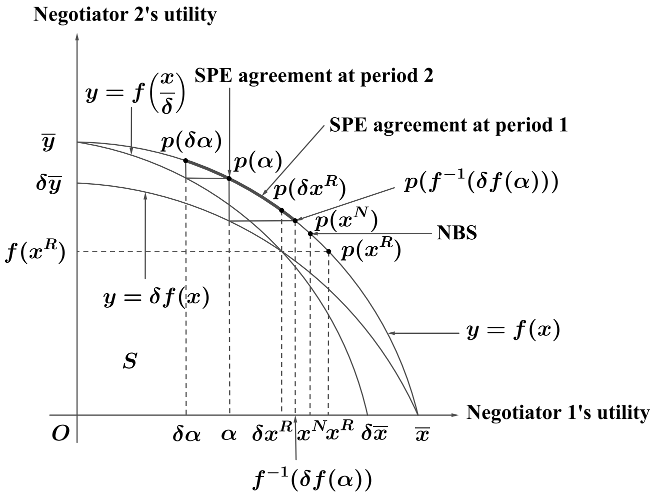

Figure 12: SPE agreement in the model with an arbitrator

the arbitrator's utility function satisfying Assumption 1. In this model, since SSPE cannot be defined, we derive SPE. Then, we obtain the following result.

Proposition 3. *The outcome that the negotiators reach an agreement with $p(x) \in \partial S$ at period 1 is supported as an SPE outcome if and only if $x \in [\delta\alpha, f^{-1}(\delta f(\alpha))]$. Also, the outcome that the arbitrator imposes $p(x) \in \partial S$ at period 2 is supported as an SPE outcome if and only if $x = \alpha$. In the SPE where the arbitrator imposes $p(\alpha) \in \partial S$ at period 2, negotiators 1 and 2 propose $x' \in [f^{-1}(\delta f(\alpha)), \overline{x}]$ and $y' \in [f(\delta\alpha), \overline{y}]$, respectively. (See Figure 12.)*

Proof. First, notice that, in all SPEs, if negotiators 1 and 2 demand x' and y' such that $(x', y') \notin S$, respectively, the arbitrator imposes $p(x')$ if $\alpha > \max[f^{-1}(y'), x']$ $(= x')$, imposes $p(f^{-1}(y'))$ if $\alpha < \min[f^{-1}(y'), x']$ $(= f^{-1}(y'))$, and imposes $p(\alpha)$ if $\alpha \in [f^{-1}(y'), x']$.

Suppose that, in some SPE σ_1, the negotiators reach an agreement with $p(x)$ at period 1 (as with Lemma 1, it is sufficient to consider the case where the negotiators reach an agreement on ∂S). Then, negotiators 1 and 2 receive x and $f(x)$, respectively. If $x < \delta\alpha$, since negotiator 1 obtains $\delta\alpha$ by deviating from σ_1 and demanding α, she can improve her payoff. Similarly, if $x > f^{-1}(\delta f(\alpha))$, negotiator 2 can improve her payoff by demanding $f(\alpha)$. Therefore, x must satisfy $x \in [\delta\alpha, f^{-1}(\delta f(\alpha))]$.

When $x \in [\delta\alpha, \alpha]$, consider the case where negotiator 1 deviates from σ_1. Then, she obtains x^* by demanding $x^* \in [0, x)$ and obtains $\delta \min\{x^{**}, \alpha\}$ by demanding $x^{**} \in (x, \overline{x}]$. Since $x^* < x$ and $\delta \min\{x^{**}, \alpha\} \leq \delta\alpha \leq x$, negotiator 1 cannot improve her payoff by deviating from σ_1. Also, when $x \in [\delta\alpha, \alpha]$, consider the case where negotiator 2 deviates from σ_1. Then, she obtains y^* by demanding $y^* \in [0, f(x))$ and obtains $\delta f(x)$ by demanding $y^{**} \in (f(x), \overline{y}]$. Since $y^* < f(x)$ and $\delta f(x) < f(x)$, negotiator 2 cannot improve her payoff by deviating from σ_1. Therefore, the outcome that the negotiators reach an agreement with $p(x)$ such that $x \in [\delta\alpha, \alpha]$ at period 1 is supported as an SPE outcome. Similarly, the outcome that the negotiators reach an agreement with $p(x)$ such that $x \in (\alpha, f^{-1}(\delta f(\alpha)]$ at period 1 is supported as an SPE outcome.

Next, suppose that, in some SPE σ_2, the arbitrator imposes $p(x)$ at period 2 after negotiators 1 and 2 demand x' and y' $((x', y') \notin S)$, respectively. Then, x must satisfy $x = x'$ if $\alpha >$

$\max[f^{-1}(y'), x']$ $(= x')$, x must satisfy $x = f^{-1}(y')$ if $\alpha < \min[f^{-1}(y'), x']$ $(= f^{-1}(y'))$, and x must satisfy $x = \alpha$ if $\alpha \in [f^{-1}(y'), x']$. If $\alpha > x'$, since negotiator 1 obtains $\delta\alpha$ $(> \delta x')$ by deviating from σ_2 and demanding α, she can improve her payoff. If $\alpha < f^{-1}(y')$, since negotiator 2 obtains $\delta f(\alpha)$ $(> \delta y')$ by deviating from σ_2 and demanding $f(\alpha)$, she can improve her payoff. Therefore, $\alpha \in [f^{-1}(y'), x']$, that is, $x' \geq \alpha$ and $y' \geq f(\alpha)$ must hold, and x must satisfy $x = \alpha$. Then, under σ_2, negotiators 1 and 2 receive $\delta\alpha$ and $\delta f(\alpha)$, respectively.

If $y' \in [f(\alpha), f(\delta\alpha))$, since negotiator 1 obtains $f^{-1}(y')$ $(> \delta\alpha)$ by deviating from σ_2 and demanding $f^{-1}(y')$, she can improve her payoff. If $x' \in [\alpha, f^{-1}(\delta f(\alpha)))$, since negotiator 2 obtains $f(x')$ $(> \delta f(\alpha))$ by deviating from σ_2 and demanding $f(x')$, she can improve her payoff. Therefore, x' must satisfy $x' \in [f^{-1}(\delta f(\alpha)), \overline{x}]$ and y' must satisfy $y' \in [f(\delta\alpha), \overline{y}]$.

Finally, we prove that, when $x' \in [f^{-1}(\delta f(\alpha)), \overline{x}]$ and $y' \in [f(\delta\alpha), \overline{y}]$, each negotiator cannot improve her payoff by deviating from σ_2. Consider the case where negotiator 1 deviates from σ_2. Then, she obtains x^* by demanding $x^* \in [0, f^{-1}(y')]$ and obtains $\delta \min\{x^{**}, \alpha\}$ by demanding $x^{**} \in (f^{-1}(y'), \overline{x}]$. Since $x^* \leq f^{-1}(y') \leq \delta\alpha$ and $\delta \min\{x^{**}, \alpha\} \leq \delta\alpha$, negotiator 1 cannot improve her payoff by deviating from σ_2. Similarly, negotiator 2 cannot improve her payoff by deviating from σ_2. Therefore, it is consistent to the fact that σ_2 is an SPE. Thus, we obtain Proposition 3. □

Proposition 3 shows that the SPE outcomes in the model with an arbitrator strongly depend on what agreement the arbitrator wishes to impose. Especially, when $\delta \uparrow 1$, $p(\alpha)$ is the unique SPE agreement. That is, the arbitrator's ideal agreement is achieved in SPEs. This result is caused by the fact that the arbitrator has the authority to decide a final bargaining outcome. Therefore, if the arbitrator is biased, the NBS is eliminated from SPE agreement. In the models of Crawford (1979) and Rong (2012), the way of arbitration is different from the above model (they use the final-offer arbitration). However, the risk by a biased arbitrator similarly arises in these models (especially when the discount factor is sufficiently large).

In contrast to it, in the model with a mediator, even if the mediator is biased, the NBS can always be achieved in SSPE. To see why this difference occurs, consider the following situation. Suppose that the mediator and the arbitrator favor negotiator 2. Then, suppose $f^{-1}(\delta f(\alpha)) < x^N$. This is simply $\alpha < x^N$ when $\delta \uparrow 1$. Also, in both models, consider the case where negotiators 1 and 2 demand x^N and $f(x^N)$, respectively. Now, notice that, when $f^{-1}(\delta f(\alpha)) < x^N$, the NBS is eliminated from SPE agreement in the model with an arbitrator (see Figure 12), but it is supported as an SSPE agreement in the model with a mediator.

In the model with an arbitrator, if negotiator 2 deviates from demanding $f(x^N)$ and demands sufficiently large value, the game proceeds to the next period and the arbitrator's ideal agreement $p(\alpha)$ is imposed. Since the arbitrator favors negotiator 2, reaching an agreement with $p(\alpha)$ at period 2 is more profitable for negotiator 2 than reaching an agreement with the NBS at period 1. Therefore, since negotiator 2 has incentive to deviate, the NBS is eliminated from equilibrium agreement.

In contrast, in the model with a mediator, even if negotiator 2 deviates from demanding $f(x^N)$ and demands sufficiently large value, the mediator does not propose her ideal agreement $p(\alpha)$ since this proposal is rejected by negotiator 1. Thus, the mediator proposes some agreement close to the NBS. For negotiator 2, accepting this proposal at period 2 is less profitable than reaching an agreement with the NBS at period 1. Therefore, since negotiator 2 does not deviate, the negotiators reach an agreement with the NBS. The case where the mediator and the arbitrator favor negotiator 1 is similarly explained. Consequently, in the model with a mediator, since the negotiators' right to reject the mediator's proposal works as a deterrent to an unfair proposal by the biased mediator, the NBS can be achieved as an equilibrium agreement even if the mediator is biased.

5 Conclusion

We considered introducing a mediator into bargaining instead of an arbitrator. An advantage of introducing a mediator is that it is easier than introducing an arbitrator since a mediator is merely an adviser. In this study, we analyzed the simultaneous-offers bargaining with a mediator and showed that the following desirable properties appear by introducing a mediator.

First, we found that disagreement is not supported as an SSPE outcome. This result implies that a mediator can resolve conflicts as with an arbitrator. Second, even if the mediator is biased, the fair agreement in the sense of the NBS can always be achieved in SSPE (an agreement having such a property is only the NBS). Therefore, in contrast to the bargaining with an arbitrator, the risk by a biased mediator does not appear. Finally, if the mediator is fair in the sense that she wishes to achieve the NBS, the negotiators reach an agreement with the NBS in SSPE when the discount factor is sufficiently large. That is, we found that the fair mediator facilitates the reaching of a fair agreement.

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